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Orthographic projection and true size in Spanish stonecutting manuscripts

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Stonecutting treatises, from the Renaissance to the Enlightenment, classify dressing methods in two broad areas. Both are named mainly after masons' tools: the square —*équarrissement, escuadría, dérobement, robos*— and the template —*panneaux, plantas*. However, the significance of these categories lies mostly in the geometrical notions they stand for; the square is applied when tracings make use of the orthographic projection, usually double, whereas the template is used where geometrical constructions provide true-size representations of the sides of the dressed stones. Thus, both methods are related to key concepts in Descriptive Geometry, the double orthographic projection for the square and developments and *rabattements* for the templates. This is not surprising, if we take into account that Gaspard Monge, the founder of Descriptive Geometry, was Professor of Theory of Stonecutting in the *Ecole de Génie* at Mézieres (Monge 1799, 4; Loria 1921:84–91; Taton 1954, 17–20; Sakarovitch 1992, 530–536; Sakarovitch 1995, 208–210; Sakarovitch 1998, 218–227; Rabasa 2000, 241).

The picture is not so clear as it appears in the treatises, however. It has been pointed out (Rabasa 2000, 158–160) that modern stonecutting mixes both methods, and that Frézier (1737, 2:14; 2:115–116), and his Spanish follower Bails (1779, 433–437) propose the dressing of some pieces by a method known as *demi-équarrissement* or *media escuadría*.

We shall deal in this paper with an early example of these hybrid methods. In *Cerramientos y trazas de montea,* a manuscript written by Ginés Martínez de Aranda around 1600, we can find stonecutting problems solved by means of pure squaring or full templates, the two canonical methods of mainstream treatises. Nevertheless, Aranda also explains the use of a two-side adjustable template, the *saltarregla* or *sauterelle*, combined with squaring; and what is more remarkable, the combination of squaring and full templates. First, we shall discuss stonecutting operations based only on orthographic projections or on true size, taking into account not only Aranda's manuscript, but also other Spanish works and, when the need arises, Philibert de L'Orme's *Premier tome de l'Architecture*; afterwards, we shall study the hybrid methods that appear, here and there, in Aranda's manuscript.

Orthographic projections

Squaring

Martínez de Aranda (1600, 113–114) includes a short, but fairly clear, description of the squaring method:

> Supongo que la figura *A* es el bolsor que quieres entrar en cuadrado y el dicho cuadrado son los cuatro ángulos *a b c d* con el cual dicho cuadrado cogerás los extremos del dicho bolsor y robándolo por el lecho alto con el robo **a** y por el tardos con el robo *b* y por la cara con el robo **c** y por el lecho bajo con el robo *d* y pasando los dichos robos

de una testa a otra quedará formado el dicho bolsor como parece en la figura *B*.

Therefore, squaring involves obtaining the orthogonal projections of the faces of the voussoir; inscribing the voussoir in an imaginary rectangular prism; cutting roughly a block with the dimensions of the prism; and taking off a number of wedges from the block with the aid of the projections, until the voussoir reaches its definitive form. Aranda does not employ the word *escuadría*, as Benito Bails (1779: 428, 430, 433) will do later; instead, he uses *robo* and *robar*, from the French *dérober*, meaning to take off or divest. However, the square allows the stonemason to transfer the wedges from one face of the block to the other; when one arm of the square rests on a flat side of the voussoir, the other arm materializes lines that are perpendicular to the flat side. In this way, the square can generate planes at right angles to the faces of the voussoir, such as the joint planes, or cylinders whose axis are perpendicular to the planes of the faces, such as the intrados surface.

This procedure has a clear geometric meaning. Architectural drawing handbooks, and even Descriptive Geometry texts, refer to the process of passing a line through a point and finding the intersection of the line with a plane, as projection. Thus, the projection of a point is a point, the projection of a straight line is a straight line, and the projection of a curve is generally a curve. This is what Aranda did when he traced the face of an imaginary voussoir. Nevertheless, in the more rigorous vocabulary of Projective Geometry, that process is a two-step operation of projection and section, and strictly speaking, the projection is only the first phase. Hence, the projection of a point is a line, the projection of a line is a plane, and the projection of a curve is, generally, a surface; thus, the intrados surface is generated as a orthogonal projection of the face arc by means of the square.

Aranda's description is purely didactic, since it involves the voussoir of a semicircular arch. In this case, all voussoirs are identical to the keystone; it is advisable to dress all stones using the projections of the keystone in order to minimize the volume of the enclosing block. Doing so, there is little difference between squaring and *panneaux*, for the horizontal projection of the intrados is the same as the intrados template.

Alonso de Vandelvira (1580, 24 v.) gives a fairly detailed description of the squaring method applied to a real stonecutting problem in the *Arco en torre cavado y redondo por robos*, that is, an arch opened in a curved wall, carved by squaring. His aim is also clearly didactic. He has explained before a more economical solution by *panneaux*, but he describes the squaring alternative «por que también sea lumbre para entender otras trazas que no se pueden hacer si no es por robos»; that is, to cast light on other problems that can be solved only by squaring. It is interesting to note that when discussing the *panneaux* solution, Vandelvira is afraid that the reader will not be quite convinced. To leave any doubt aside, he advises to make a model of the arch by squaring: «si lo quisieres probar contrahaz un arco de éstos por robos, como te enseñaré adelante, y luego planta estas plantas y harás la prueba ser éstas ciertas». Thus, Vandelvira accepts as an empiric proof the simple squaring method, rather than to the complex *rabattements* of the *panneaux* method.

According to Vandelvira, «después de haber trazado su arco y torre cavado y echados sus plomos, pondrás las piezas en cuadrados desde las tardosas a las mochetas, así en el arco como en el grueso de la pared del torre cavado». That is, the squaring method requires a tracing, usually made in full scale on a floor or a wall; but the tracing is relatively simple, compared with the more elaborate operations necessary to employ the *panneaux* method. It will be sufficient to construct a plan and an elevation of the arch, to divide it in voussoirs, and to trace an enclosing rectangle around each voussoir, both in plan and in elevation.

The first operation of the stonecutting process is to carve a block with the dimensions of the enclosing rectangles. In the next step, the stonemason should take off four or five wedges from this basic block to give the voussoir its final shape. There are small differences with the procedure explained by Aranda. In an ordinary voussoir, the mason should take off the wedges of both joints, that of the intrados, and two additional wedges, corresponding to both curved faces. In the first voussoir or *sommier*, there is no need to take off a wedge for the low joint, given that it is horizontal. As usual in Spanish stonecutting, nothing is said about the extrados, either in the ordinary voussoir or in the *sommier*.

However, it is striking that Vandelvira advises to carve the curved faces in the first place: «lo largo su

torre cavado y redondo . . . lo cual se ha de robar primero que nada». Doing so, the stonemason can dress those faces with the aid of a square leaning in the horizontal upper plane of the block, since both curved fronts are cylinders with vertical generatrices. It is not easy to understand how the stonemason will carve both joints and the intrados. According to Vandelvira, the intrados should be dressed «con la cercha del arco echándola por cuadrado»; that is, with a one-sided template or *cerce*, keeping it squared. Looking at the tracing, it becomes evident that the *cercha* represents the orthographic projection of the arch on a vertical plane perpendicular to the axis of the arch. Since Vandelvira makes no attempt to develop the arch, then «echándola de cuadrado» means keeping it parallel to this vertical plane. However, this is not easy, for this plane is parallel to the planes of the front and rear sides of the block; and these sides were suppressed when the curved faces were dressed before the intrados, following Vandelvira's explicit instructions.

In my opinion, the simplest solution to this riddle lies in Aranda's remark that the *robos* should be brought from one face to the other. In our case, the stonemason must carve the basic block and mark on it the shape of the face, obtained by orthographic projection. After that, the mason can make four straight linear courses or *tiradas* joining the vertices of the orthographic projections of the arch in both faces, still planar, before dressing the curved fronts. After carving the curved fronts, the stonemason can easily dress the joints with the aid of a ruler leaning in two *tiradas*, since all straight lines that intersect two parallel lines are in the same plane. More care is needed in the dressing of the intrados; that is why Vandelvira states that the *cercha* should be kept *de cuadrado*, that is, orthogonal to both *tiradas*. Doing so, the *cercha* is parallel to the projecting plane, since these *tiradas* are orthogonal to the vertical plane of projection, and two planes that are orthogonal to a straight line are parallel to each other; in this way, the movement of the *cercha* generates the intrados cylinder.

Projected templates

Of course, the dimensions of the block can be transferred to the stone by means of a gauge or any measuring instrument, and there is no need to use templates in this step. If the definitive shape of the voussoir is fairly simple, it can be brought to the stone in the same way; this seems to be the case in Vandelvira's explanation. However, stonemasons employed templates to transfer complex voussoir faces, even in the context of the squaring method (Frézier 1737, 2:12–13; 2:108–109; Pérouse de Montclos 1982, 90; Palacios 1986, 102). It is interesting to note that for Martínez de Aranda the templates used in the *panneaux* method are *plantas al justo*, which can be translated as «exact templates»; but when templates are used in connection with the squaring method, the operation is called *plantar de cuadrado*, a word that means *orthogonal* in sixteenth-century masons' jargon, and comes from the same root as *escuadra*, *escuadría, équerre, équarrissement* and *square* (Martínez de Aranda 1600, 7, 12, 85).

True size

Templates

Vandelvira and Aranda explain how to construct true size templates in the majority of the *trazas* of their respective manuscripts. Leaving aside flexible templates, and those that try to represent a warped quadrilateral, these methods are based either on an antecedent of the *rabattements* of Descriptive Geometry or on triangulation. Both authors give fairly similar descriptions of the tracing of the *plantas* in the *Arco viaje contra viaje por testa*, or simply *Viaje por testa*, a skew arch (Vandelvira 1580, 19 v.; Martínez de Aranda 1600, 9–11). After tracing the arch in plan, the mason should construct an elevation with the projection plane orthogonal to the intrados joints. Since the arch is biased, the picture plane cannot be parallel to the arch face. To construct the intrados template, the stonemason should rotate the intrados of a voussoir around the lower intrados joint; of course, this joint will not move in the rotation. Since the upper intrados joint is parallel to the lower joint, it will stay parallel after rotation; besides, he can read the distance between both intrados joints from the elevation, for both are perpendicular to the projection plane and are projected as points; in this way, the stonemason can trace the upper intrados joint. To find the ends of this joint, he can take into

account that both move in a plane that is perpendicular to the axis of rotation; the horizontal projection of this plane will be a line that passes by the projection of the end of the joint and is orthogonal to the lower intrados joint; the end of the joint shall be where this line intersects the upper intrados joint. In this way, the mason can trace the intrados template, knowing its four vertices.

However, this method makes the face arc elliptical and the cross-section of the arch semicircular. For aesthetic reasons, from the 16th to the 18th centuries, architects and stonemasons usually preferred arches with semicircular faces and elliptical cross-sections (Frézier 1737, 1:279; Rabasa 1994, 147; Rabasa 2000, 304). De L'Orme struggled to explain this solution in an obscure passage (1567, 67 v. – 69 r.) that suggests the problem was addressed by means of squaring.

Vandelvira (1580, 28 r.) tried to solve the problem constructing an elevation so that the projection plane is parallel to the face plane; this allows him to trace easily the semicircular face arc. The trade-off was that the intrados joints were not orthogonal to the projection plane, and that prevented Vandelvira from using the simple rotation technique he had used in the *Viaje por testa*. To overcome this difficulty, he resorted to a triangulation technique, constructing the intrados template from the lower intrados joint, that is horizontal and, hence, represented in true size in the horizontal projection. To obtain an end of the upper intrados joint, he constructs a right triangle whose catheti are the horizontal projection of the diagonal of the intrados template and the difference in heights of both ends of the diagonal; of course, the hypotenuse is the length of the diagonal in true size. Vandelvira traces then two arcs, one with centre in one end of the lower intrados joint and radius equal to the length of the diagonal and other with centre in the other end of the lower intrados joint and radius equal to the length of the chord of the face arc. The intersection of both arcs gives one end of the upper intrados joint. The other end of the upper intrados joint can be placed using the same construction; that allows tracing the intrados template. Though this construction is clever, it is also slow, recursive and prone to accumulate errors. Ginés Martínez de Aranda, Alonso de Guardia and, probably, Cristóbal de Rojas use a different construction based on rotations to solve the problem, but we shall deal with it later (Cristóbal de Rojas 1598, 99 v.; Martínez de Aranda 1600, 16; Alonso de Guardia 1600, 80 v.; Calvo 1998, 69–70).

None of the Spanish manuscripts includes a full explicit description of the stonecutting process using these templates. According to Philibert De L'Orme (1567, 99 r.):

> pour les paneaux de ioincts, paneaux de teste, & aussi paneaux de doile par le dessus, gardez vous bien de les trasser pour coupper la pierre du premier coup, car vous la gasteriez, & ne pourroit plus seruir. Il fault doncques oster vn peu de d'vn des ioincts, & puis vn peu du costé de la teste, semblablement du costé de la doile de dessus, . . . & non point tout à vn coup, mais couppant si dextrement le tout que vous puissiez armer vostre pierre de paneaux tout autour qui se rapportent iustement & se touchent l'vn l'autre par toutes leurs extremitez, tant que par les ioincts que par les doiles & par le deuant, ou est le paneau de teste

Normally, the use of pure templates is not as difficult as it appears from De L'Orme's colourful description. Philibert is talking about a *trompe*, a cut that involves a specific difficulty in the acute vertex of the triangular shape of the intrados. Applying his method to the voussoir of an arch or vault, an ordinary stonemason can dress a flat face in the intrados and inscribe the shape of the intrados template on it. After that, he can start gradually taking stone off from two adjacent sides of the voussoir, say the front and the upper joint, shaping two planes that pass by the intrados joint and by the chord of the face arc. It is important to check the result at intervals by means of the corresponding joint and face templates, as they approach the position in which both templates assemble with each other and with the intrados template. When this point is reached, the face joint is fixed in space, and the planes of the face and the joint are also fixed; the stonemason can dress both easily with the help of the ruler.

To our ears, this procedure sounds inneccesarily complex and dangerously empiric. In the 18th century, Frézier (1737, I:372–374), addressed the problem in a more efficient way, using the dihedral angle between the intrados and joint planes; that is, the angle between the intersections of both planes with a third plane that is orthogonal to their common intersection, in this case the intrados joint. Apparently, this idea is too abstract for De L'Orme or the Spanish manuscripts of the 16th century, for none of them mentions it. However, the subsequent steps in the

dressing process we are describing are much simpler. Once the planes of the face and the joint are dressed, the stonemason can in turn inscribe on them the corresponding templates. After this, the mason can dress the plane of the lower joint with the aid of a ruler leaning in the intrados joint and the face joint, inscribe the joint template on it, and dress the rear face making the ruler rest on both face joints and the chord of the face arc.

As we have said before, no Spanish manuscript or treatise describes this method explicitly. However, Martínez de Aranda explains the construction of intrados and joint templates in almost each *traza* of his manuscript. Face templates are usually present, either included in the elevation belonging to the tracing, or either by means of the construction of the *cimbria*, that is, a set of face templates. This suggests strongly that Aranda considered this procedure as the canonical method in stonecutting. Perhaps for this reason he never explains it explicitly, while he remarks the details that diverge from this paradigm, as we shall see below.

Templates and saltarreglas

Notwithstanding that, some evidence suggests that the most widely used dressing method in 16th Spain was a simpler one. Alonso de Vandelvira usually explains the construction of the intrados templates, calling them simply *plantas*, but not the joint templates, the *plantas por lecho* of Martínez de Aranda. Instead, he constructs usually *saltarreglas*, that is, lines that represent the face joint and allow him to measure the angle between the intrados joint and the face joint; the *saltarregla* takes its name from the protractor using by the stonemason to transfer these angles, from the French *sauterelle*. Some interesting details make clear that the ensemble of these *saltarreglas* and the intrados joint plays the role of a simplified joint template. In his *Viaje por testa*, Vandelvira (1580, 19 v.) explains the construction of *plantas* and *saltarreglas*, adding that «si quisieres echar molduras has de extender los moldes en las saltarreglas». The result is the most complete joint template one can think of, representing not only its four edges, but also a highly detailed section of the moldings. On the other hand, in many *trazas*, Aranda (1600, 16, 19, 25, etc.) represents the three edges of the joint template corresponding to the intrados joint and both face joints in solid line, while he renders the extrados joint in dashed lines. This graphic treatment seems to let the reader choose between applying the intrados and face joints as *saltarreglas*, and using four joints as a full template.

Vandelvira gives also few hints about the way *plantas* and *saltarreglas* are used. However, the consideration of the *saltarregla* as a simplified template suggests a variation of De L'Orme's method. After dressing the plane of the intrados side of the voussoir and marking on it the intrados template, the stonemason can gradually take material from the face side of the voussoir, until the face template and the protractor, opened in the angle marked by the *saltarregla*, assemble in the face joint. This method is less cumbersome than the use of joint and face templates, for the protractor can be used even if the joint is not dressed yet. Once the first face joint is fixed in space, stonecutting can go on as we have explained before. Inversely, Alonso de Guardia (1600, 82 v.) explains the dressing of a voussoir of the *Arco abocinado* using an intrados template, a joint template and the *baivel* or bevel of the arch.

Hybrid methods

Orthographic projection and rotation

Squaring is not economical, neither in material nor in labour (De L'Orme 1567, 73 v.). Renaissance stonemasons tried to find methods, still based in orthogonal projections, that would reduce this loss. One of these consists in inscribing the voussoir, not in a rectangular prism with horizontal and vertical faces, but in a mixtilinear prism that circundates closely the projection of a voussoir in a vertical plane.

In the *Arco capialzado por robos*, Martínez de Aranda (1600, 40–41) explains the carving of the voussoirs of an arch with a semicircular arch in the front face and a segmental arch in the rear. As in other occasions, prior to the dressing of the voussoirs, the stonemason is to make a full-size tracing. This tracing is fairly simple, for it involves the construction of the plan of the arch; the elevation, with the semicircular and segmental arches; and the division of the arch in voussoirs, by means of a set of planes that pass by the axis of the segmental arch. It is interesting to note that

Aranda does not represent the plan of the intrados joints, for he will not use their horizontal projection in the carving process.

The lines crossing the intrados in the plan are not projections of the intrados joints, but rotations of them, constructed with the aim of measuring the angle between front joints and intrados joints; this rotations of the intrados joints are called *saltarreglas* by Aranda. It is important to take into account that Vandelvira gives that name to face joints, not intrados joints, traced also to measure the same angle. Thus, the word *saltarregla* seems to be associated to the angle between both joints; that is not surprising, for the *saltarregla* is in the first place the stonemasons' protractor or *sauterelle*.

To construct the *saltarregla*, Martínez de Aranda rotates the line 1 5 around a horizontal line orthogonal to the projection plane, that is, the plane of the arch face. In this way, he brings the line 1 5 to a horizontal plane, to measure its angle with the front joint. In this rotation, the point 1 does not move, since it belongs to the axis of rotation. The point 5 will move in the plane of the arch face, since this plane is perpendicular to the axis of rotation. Besides, the distance from 5 to the axis of rotation can be read from the elevation, since the axis of rotation is perpendicular to the plane

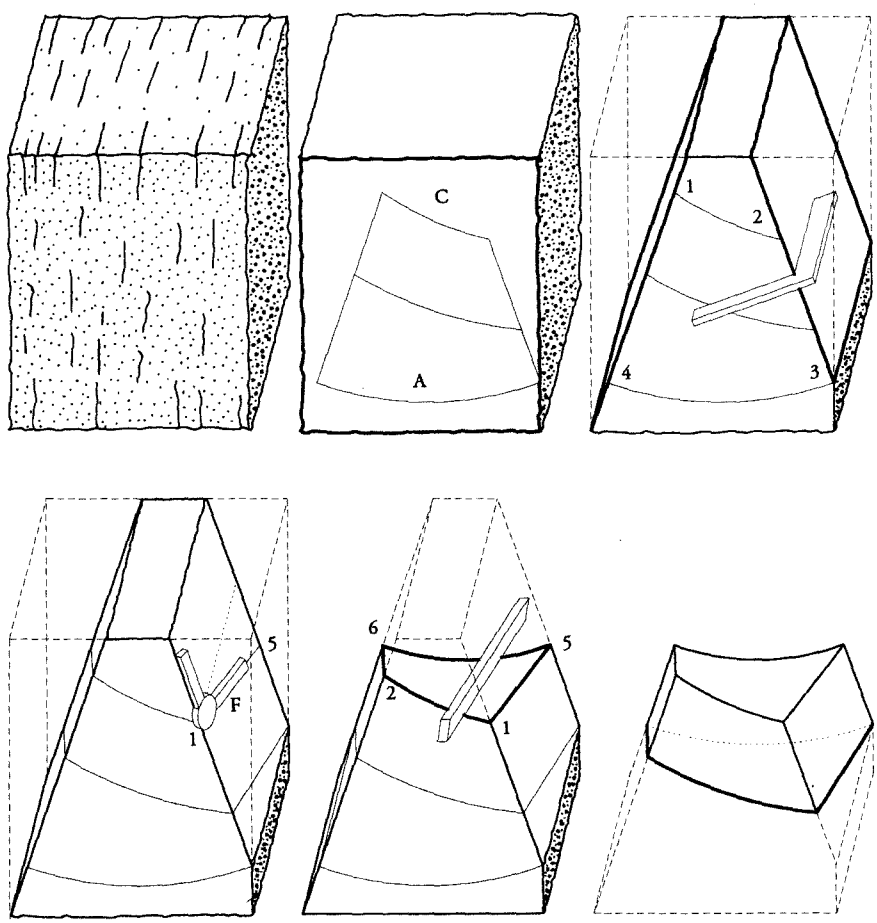

Figure 1
Dressing of a voussoir of the *Arco capialzado por robos*

of projection and therefore it is projected as point 1. Transferring this distance to the plan from the axis of rotation, Aranda can place the point 5 after rotation; joining it with 1, he can construct F, which represents 1 5 after rotation. As for the face joint, it will move in the face plane; so it will be represented by the horizontal projection of the arch face. Repeating the operation for the other side of the voussoir, he can construct G, representing 2 6 after rotation.

Once the tracing is done, the stonecutting process is fairly simple, and much more economic in labour and material than ordinary squaring. The stonemason starts by carving a mixtilinear block with the base in the shape of 1 2 3 4, and height equal to the distance between B and D. After that, he can mark the angle between the intrados and the front joints in the joint side of the block, with the aid of the *saltarregla* or protractor. Repeating the operation on the other side, he is able to place the points 1 2 5 6, that define the four vertices of the intrados face of the voussoir, and to take material from the block until the intrados face is correctly dressed. In the *Arco capialzado viaje por cara,* which is a biased variation of this arch, Martínez de Aranda (1600, 46) suggests the use of a ruler to check the dressing of the intrados, stating that the ruler should create a non-developpable ruled surface: «de unas testas a otras las labrarás a regla plantando la regla de cuadrado que vengan a quedar por las caras engauchidos«.

Orthographic projection and true size

Using the same technique in more complex pieces, Aranda arrives gradually at the combined use of templates and squaring. In the *Arco por arista en la cara,* Martínez de Aranda (1600, 46–47) addresses the problem of an arch formed by two *arcos capialzados* joined back to back; that is, an arch with a «V» section, with two semicircular faces and an *arista* or groin in the form of a segmental arc, on an vertical plane parallel to the planes of the two faces. As in the preceding example, Aranda starts by making a simple tracing, that represents the plan of the arch with both faces and the intermediate groin; the elevation with the semicircular fronts and the segmental *arista*; and the division of the arch in voussoirs. To construct a schematic joint template of each voussoir, he rotates both segments of the V section around a line that is orthogonal to the faces of the arch and passes through the intersection of the *arista* and the plane of the joint. This point is on the rotation axis and will not move. Point 5 and its counterpart in the other face will move in the face planes; he can take their distances to the axis of rotation from the elevation and bring them to the plan; joining the two points with 1, he can trace a simplified joint template in true size. Of course, this is no more than a duplication of the method Martínez de Aranda had used in *Arco capialzado por robos,* but instead of a single *saltarregla*, the result is a template representing the «V» cross-section.

The description of the stonecutting operations hints strongly that the method is transitional between squaring and templates. According to Aranda, «la labraras primero de cuadrado con la forma que tuviere entre los cuatro puntos 1 2 3 4 que tenga de grueso lo que tuviere de ancho la planta del dicho arco»; that is, the stonemason should carve a mixtilinear prism, with the width of the arch and the shape defined in the elevation by the projections of the extrados, the groin, and the two joints. After this, he should take off the wedges that are below the ruled surfaces that pass by a face arc and the groin; to do so, Aranda advises to mark in both joint sides of the block the shape of the «V» cross-section of the arch. This section is not a single line, as in the *Arco capialzado*; hence, Martinez de Aranda designates it as *planta por lecho*, or joint template, while using the traditional word for squaring: «la robarás por entrambas testas con el robo que parece entre los números 1 2 5 6 que venga a quedar después de robada por el lecho bajo con la forma que tuviere la planta por lecho F y por el lecho alto quedará con la forma que tuviere la planta por lecho G».

True size and orthographic projection

In the *Arco por arista en la cara*, the stonecutting procedure is based mainly on orthographic projections, but Aranda uses a true size template as an auxiliary device. Inversely, in the second *Arco en viaje por cara y por plantas* of the *Cerramientos and trazas de montea*, the dressing of the voussoirs is carried on with the aid of full size templates, but Martínez de Aranda (1600, 15–16) advises to use the

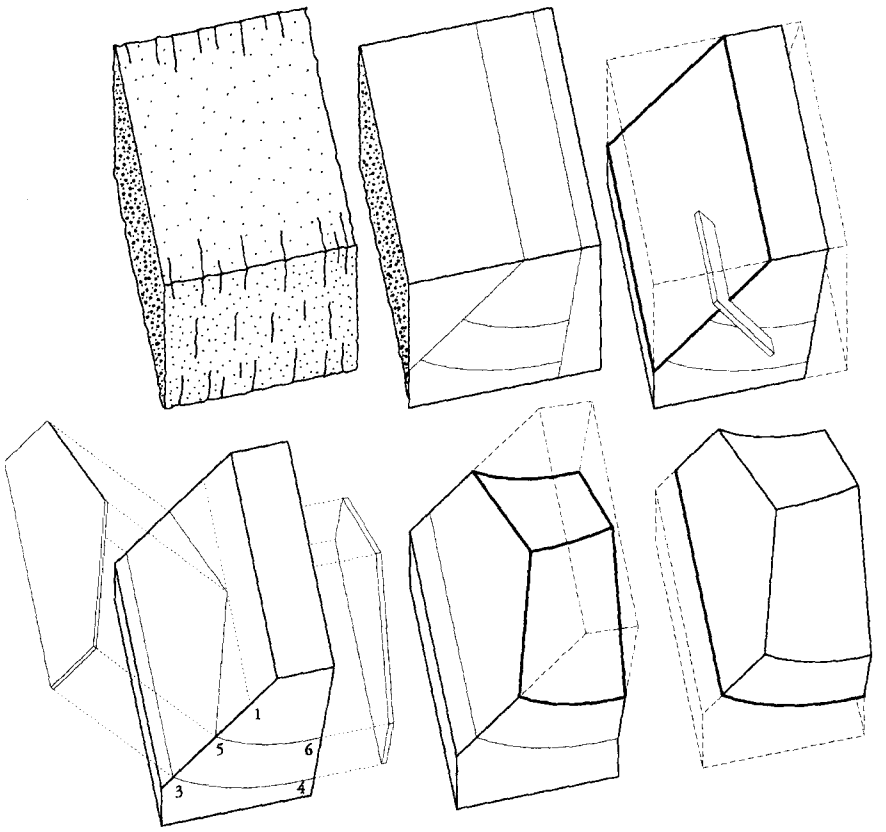

Figure 2
Dressing of a voussoir of the *Arco capialzado por robos*

projection of a *cercha*, that is, a one-sided template, to carve the intrados of the voussoir.

As always, the stonemason is to make a full scale tracing, in plan and elevation, this time of a skew arch. After that, the mason should trace true size templates of the joints and the intrados of each voussoir. As we have remarked before, Aranda uses here an antecedent of the *rabattements* of Descriptive Geometry, to avoid the disadvantages of Vandelvira's triangulations. The construction relies on a property of rotation: when rotating around an axis, a point will move on a perpendicular plane to the axis of rotation. Thus, when constructing an intrados template, Aranda makes it rotate around the lower intrados joint, 4 8. The points 4 and 8, that are in the rotation axis, will not move, but the edge of the upper intrados joint, 5, will move on an orthogonal plane to the lower intrados joint, and its horizontal projection will be on a perpendicular line to the axis of rotation. Besides, we can take the distance between points 4 and 5 from their vertical projections 3 and 2, since both are in a frontal plane, that of the face of the arch. That allows the stonemason to trace an arc with centre in 4 and radius equal to the distance between 2 and 3; the intersection of this arc with the perpendicular line will be point 5 in the template. The mason can construct point 7 in a similar manner, or take into account that the line 5 7 is parallel to the axis of rotation and will be represented parallel to 4 8 in the template; therefore, the quadrilateral 4 5 7 8 will give the intrados template.

Aranda advises to use the same method for the

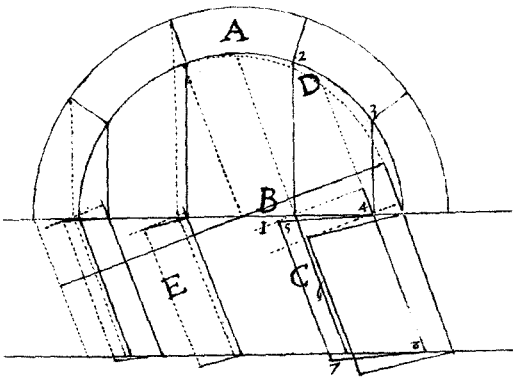

Figure 3
Ginés Marínez de Aranda. *Cerramientos y trazas de montea*, pl. 16. *Arco en viaje por cara y por plantas*

construction of the joint templates. Additionally, the stonemason can make face templates, taking them directly from the elevation of the arch, since Aranda says that «hánse de labrar las piezas por las testas con la forma que tuviere el arco semicírculo A»; that is, the voussoir should be cut by the faces with the form of the semicircular arch A. In this way, when the mason starts to dress the voussoirs, he has an intrados template, two joint templates and, if necessary, two face templates. This seems more than enough to employ the procedure suggested by De L'Orme, enclosing the voussoir in the five templates and cutting it gradually until all templates match.

However, Aranda instructs the stonemason to trace the cross-section of the arch, by a vertical plane perpendicular to the arch axis. The procedure resembles closely the *changement de plans* of Descriptive Geometry. First, the mason should trace the horizontal projection of the new vertical projection plane, in the manner of a folding line; second, he should trace reference lines, perpendicular to this folding line, passing by relevant points in the cross-section; third, he should transfer to this reference lines the heights of the relevant points, taken from the elevation. The cross-section is different from the face arc, due to the bias of the arch; in this case, since the face arc is semicircular, the cross-section is a raised half-ellipse or *arco encogido*. Once this is done, the mason can follow Aranda's advice: «por las caras de los bolsores se han de labrar de cuadrado con la forma que tuviere el arco encogido tirado en blanco D»; that is, the intrados of the voussoirs should be dressed squarely with the shape of the raised ellipse. That can be done with a *cercha* or one sided-template that is carried along the intrados, keeping it parallel to the plane of the cross-section; that is, orthogonal to the intrados joints. Doing so, the intrados surface, an elliptical cylinder, is generated as the projection, in the strict sense of the word, of the *arco encogido*.

Oblique projection and true size

Aranda (1600, 47–48) includes in his manuscript a skew variation of the *Arco por arista*, called *Arco por arista en la cara en viaje*. He starts tracing a doorway as a semicircular arch; but since the plane of the doorway is oblique to the axis of the arch, the extrados surface will be an elliptical cylinder. As in

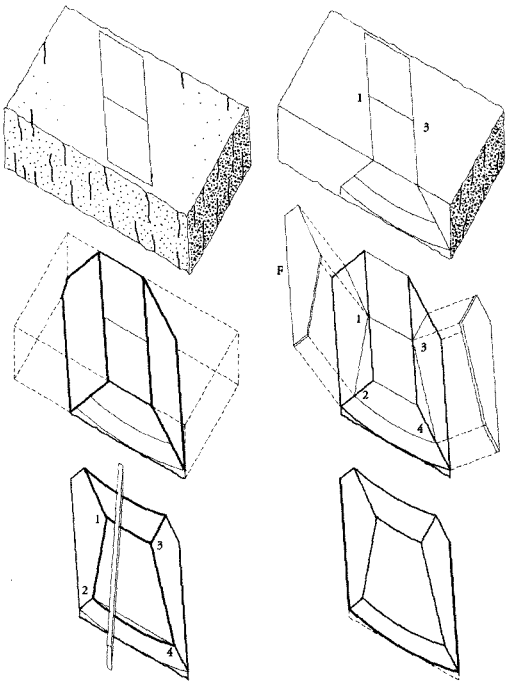

Figure 4
Dressing of a voussoir of the *Arco por arista en la cara en viaje*

the *Arco por arista en la cara*, the intrados is formed by two ruled surfaces that pass by a face arc and a groin. The elevation is not an orthogonal projection, but an unusual oblique projection. The picture plane is parallel to the arch faces, but the projecting lines are parallel to the axis of the arch; hence, they are horizontal, but biased in relation to the projection plane. In this way, both face arcs, and even the springing of the groin or *arista*, are superimposed in this strange elevation.

Aranda starts the stonecutting process by carving a block defined by points 1 and 3 and the extrados joints. However, this gives only the face side of the voussoir. He cannot carve the voussoir *de cuadrado* as he could in the previous examples, for the joints are not orthogonal to the face planes. Therefore, to pass the face template from one face to the other, as Aranda did in his theoretical explanation of the *robos* method, he needs the *planta por cara* or intrados template of the block. According to the manuscript, the stonemason should construct the intrados template by means of the procedure we have seen in the preceding section of this paper. On top of this, he will also mark the line *arista* or groin in the template and transfer it to the stone. The *planta por cara* is then a real template that represents the true size and form of the intrados of the intermediate block from which the voussoir will be carved, not of the intrados of the definitive voussoir.

When tracing the *plantas por lecho*, or joint templates, the stonemason is to construct first the joint template of the enclosing block, using *rabattements* as before. Since the *planta por cara* does not represent the definitive voussoir, it is also necessary to take away two wedges, below the ruled surfaces that pass by the face arcs 4 2 and the groin 1 3. To do that, he can take from the oblique elevation the distance between the projection 1 of the groin and the projection 2 of the face arc; transferring this distance to the representation of the face joint in the *planta por lecho*, he can get a corner of the joint template of the shaped stone. Aranda's text is short but unambiguous: «en las plantas por lechos formarás segunda vez las plantas por lechos para plantarlas al justo como parece en las plantas por lecho **F** y les robarás las piezas conforme se hizo en el arco por arista en la cara». That is, the mason is to construct a second V-shaped joint template to cut the voussoir to its definitive form, robbing it as in the *Arco por arista en la cara.*

Conclusion

Due to reasons of space, I have not dealt with two interesting topics closely related to these methods: the problems posed by flexible templates, that are related to developments of surfaces, rather than true size of planar figures; and that of non-developpable surfaces, which I have discussed in a previous paper (Calvo 2002). Since my exposition focuses mainly on *Cerramientos y trazas de montea*, and the fourth and fifth parts of this manuscript, dealing with vaults, are lost, all the examples discussed here are arches. I have left aside vaults and a stonecutting instrument so characteristic as the *baivel*, which was used mainly on vaults and *trompes*.

Nevertheless, I hope that these examples are sufficient to illustrate some basic points. Spanish stereotomic manuscripts of the 16th century, as French treatises of the period and the subsequent centuries, prefigure many central notions of Descriptive Geometry. They not only employ orthographic projection with remarkable ease, but also templates in true size, constructed by procedures that are conceptually similar to modern day *rabattements*, rotations and auxiliary views or *changement de plans*.

If the discussion of squaring and templates in the treatises of Derand (1643, 18–21) or Frézier (1737, 2:11–15) is akin to a Descriptive Geometry textbook, their use in Vandelvira or Aranda reminds more of a workbook or a collection of exercises. Martínez de Aranda only deals with general principles on one occasion (1600, 113–114) and even then only by means of an example. Of course, Aranda does not use the modern word *proyección*, and designates the horizontal projection as *plomo*, that is, «plumb line». When he needs a word for the vertical projection, he has to resort to a term so far apart from the masons' jargon as *imaginación* (Martínez de Aranda 1600, 85).

The lack of conceptual discussions contrasts with the ease that Vandelvira, and especially Aranda, show when combining the most suitable tools to solve a particular problem, whether orthogonal or oblique projections, rotations, *rabattements*, or triangulation. This strongly suggests that squaring and templates, *équarrissement* and *panneaux*, *robos* and *plantas*, are mainly didactic categories. Though they were present in the jargon of ordinary masons, as the titles of many

trazas make clear, they were not incompatible, and in everyday's work, masons probably combined them freely, as Rabasa has remarked about modern stonemasons (2000, 158).

As stated at the beginning of this paper, stereotomy has been held to be a forerunner of Monge's double-projection system. Furthermore, the examples discussed here show that the role of stonecutting in the development of descriptive geometry can be approached from a wider perspective. A secular practice, dating at least from the 16th century, employed in their everyday tasks by Spanish and French stonemasons, furnished Monge and his followers not only with the method of double orthogonal projection, but with a whole **integrated** system of geometrical problem-solving tools, including rotation, *rabattements* and even *changements de plan.*

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