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Shells and membranes in the architecture of the XVIIIth century

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The Baroque has been traditionally a style minimized by its tendency to the decoration and its lack of systematic. Gradually it has left demonstrating that this was a prejudice and that the architectural values were of first magnitude, comparable to the biggest landmarks of classic styles. As for the structural aspects we want to demonstrate that they overcome to as much as it was made previously by their space wealth, their economy of means and the intelligence of their solutions. Somehow they represent an antecedent of the shell architecture that so much it would make an exhibition with the Reinforced Concrete and the analytic techniques of dimensioning. With less means but with great intuition, the Baroque, invented forms of a complexity that today is still difficult to understand, at least geometrically. Paradoxically there are not investigations that help us to clarify the structural miracle of the Baroque since up to now it has been in historians' hands and not of architects or engineer's. This article tries to open road in this unexplored field.

Introduction

To speak of shell structures in the mathematical and modern sense of the term, in designs previous to 1900 and in materials different to the reinforced concrete it seems an excessive license.

However, since in the XIX century the brick vaults already demonstrated a good capacity to adopt capricious forms in absence of flexions and with the brick and the mortar like only materials. It is logical to think that the construction of this shells is previous to their analytic tools, so previous justification that it can be extrapolated to the Byzantine construction in that churches like Serlio and Bacchus or Sta. Sofía, they have been able to be designed like membranes, with great precision if we compare them with the results obtained by Finite Elements.

However the first properly shells are not found until the Baroque architecture, and not in fact in the Italian where architects like Brunelleschi, Miguel Ángel or Palladio did great achievements to overcome the structural challenges of the antiquity. It is later, when the economy of means that the XVIII Century imposed to the construction, when a great quantity of apprentices, that were formed in San Pedro's factory, were disseminated by Europe to create some national styles, rich in way, splendid of concept and good structurally.

The Baroque structures

The new generation that was in Italy, following Miguel Angelo and Borromini guidelines, preferred to materialize their structures in nerves and complex laces very of the medieval style. Guarini (Fig. 1a) or Vittone (Fig. 1b) they go in this line.

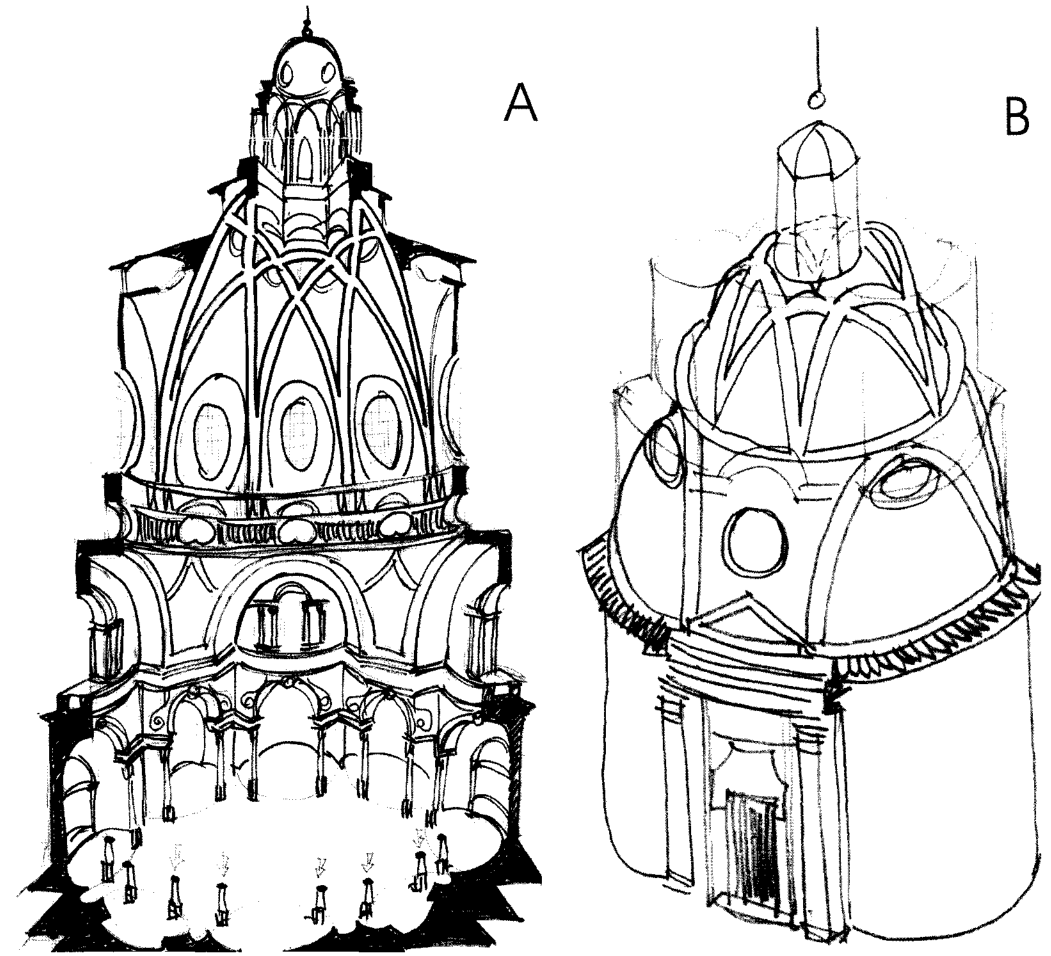


Figure 1a and 1b
St. Laurence in Turin by Guarino Guarini. - St. Bernardino in Chieri by Vittone

Who moved to the north they preferred to still use continuous forms more complex than the mentioned nerves, although masked under a dense ornamental layer of stuccos and paintings. Fischer von Erlach, patriarch of all them, in Austria, the family Dientzenhofer in Prague and Neuman in Germany are some few ones of among the dozens of architects that we could mention that, with its differences they dedicated to create suggestive and complex spaces with minimum structures.

While Fischer was devoted to explore the elliptic spaces (Fig. 2) and the Dientzenhofer family the intersected geometries (Fig. 3), Neuman manages with ease the most difficult spaces, mixture of ellipsoids and cylinders on punctual supports (Fig. 4a). Zimmerman gets geometries that don't have recognisable forms (Fig, 4b).

When we begun the study on the structures of the Baroque, we were surprised by the lack of bibliography and of specific studies. It is accentuated it in the case of the Central European Baroque, in the which, apart analysis made for consolidation works and repair or reconstruction, doesn't exist practically any research. We except Jager and Croci although they have not

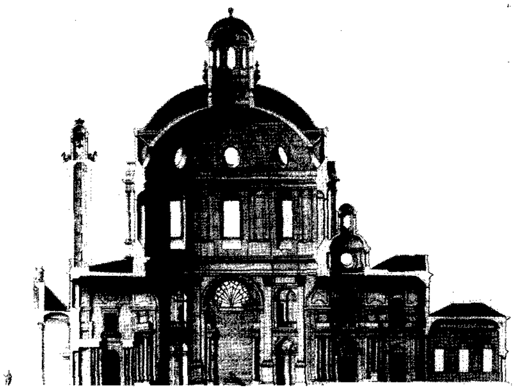


Figure 2
St. Charles in Vienna Fischer von Erlach

intervened in particular aspects of shells. It is for it that we contribute the data that continue with intention of to open the road and to demonstrate the great structural interest that has a recently valued style.

The work is wide and it will embrace the different types of Baroque membrane. But at this time it is logical to begin with important and very well-known monuments, like they can be:

1) Sanctuary of Kappel near Waldhsassen of George Dienzenhofer (Fig. 5a).
2) Church of pilgrimage of Steinhausen of Zimmerman (Fig. 5b).
3) Franciscan church of Verzhenheiling, by Neuman (Fig.6a).

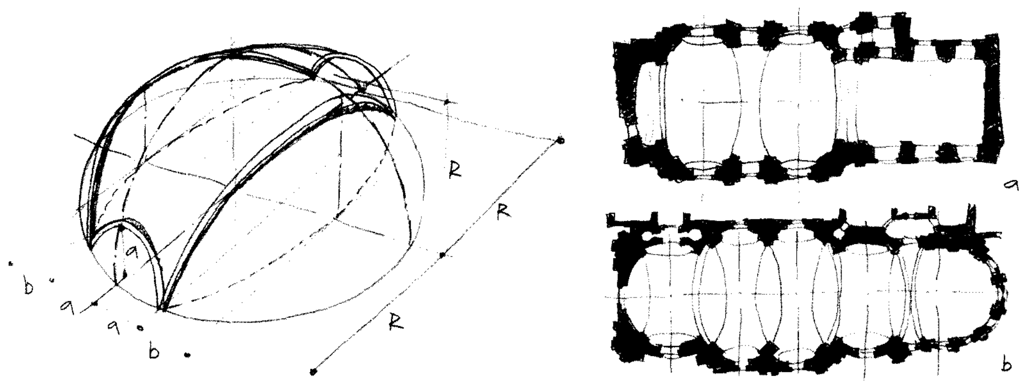
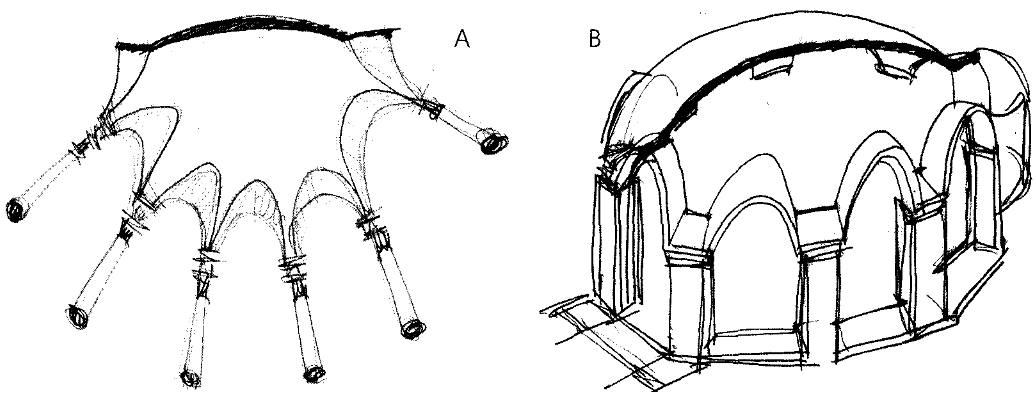
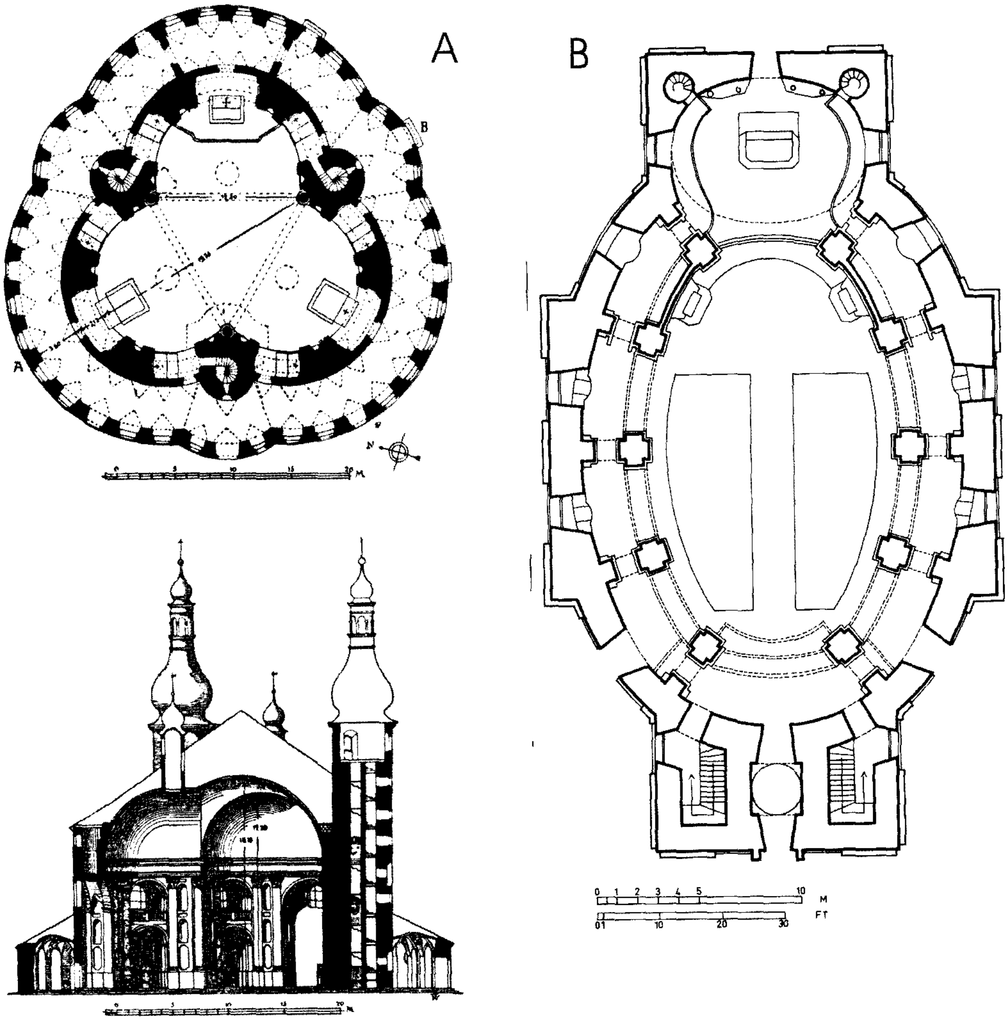


Figure 3
Selection parts of a sphere to build the vaults by George Dientzenhofer



Figures 4a and 4b
Type of Neuman Construction - Zimmerman's Steinhausen Church



Figures 5a an 5b
Sanctuary of Kappel. - Church of Pilgrinage of Steinhausen

4) Benedictine church of Neresheim, by Neuman (Fig. 6b).

In all these cases we will make the same suppositions:

a) For lack of data we consider that the vaults are built with bricks to thread and therefore with a thickness of 0.25 m.

b) The characteristics of the fabric are the following ones (Hydaka 1989):
   1. $\gamma = 1.7$ Ton/m^3
   2. E = 10 E^6 Ton/m^2 and the material is elastic and lineal in the considered range.
   3. $\gamma = 1/6$

c) The vaults are supported by hinges on the cornices and only the part upon them are

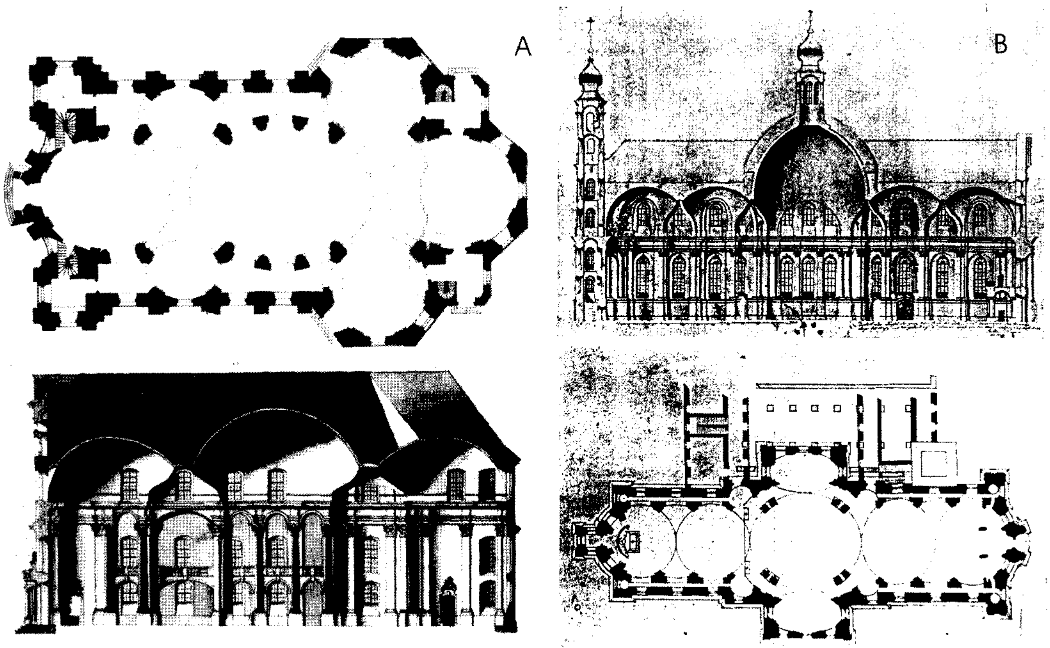


Figure 6a and 6b
Church of Vierzhenheiling. - Benedictine church of Neresheim

calculated starting from this level. It is necessary to keep in mind that the cornices, in the Baroque, define the form in plant and that what there are under them is well rested.

d) The only actions that we will consider they are those of own weight.
e) The mathematical pattern that we will use is to consider the medium surface of the vault as the geometry for analysis.
f) The method used to calculation is that of Finite Elements. The resulting graphics will represent the efforts and displacements in the considered surface.

Sanctuary of Kappel in Waldsassen. By George Dientzenhofer

In this case we have a estrange functional distribution with a triangular plan with curved sides, according to the geometric outline of the Fig. 7. It has odd number of sides, with solid walls perforated with chapels and stabilizing towers.

The dome is formed by a sphere that intersects with other three, and the group is supported in all its contour. There are not more perforations than some small chimneys that provide a soft clarity (Fig. 7).

George Dientzenhofer was able to solve in this small alpine chapel a temple of centralized plant that, in certain way, it was inspired by the Roman Panteon, with a certain form classicism but with the disturbing triangular form. Something that we will be for the first time in the Baroque and that gives certain keys of the style, it is the Ottoman reminiscence. In 1683 the Turks were about to take Vienna and, although they failed, this opened up a period of relationships diplomats with the Sublime Door, and, there were probably scientific exchanges and technicians. The hexagonal Ottoman plants gave cause to this triangular construction, as the Mosque of Serefeli and also Sta. Sofía or the Mosque of Soleiman in Istanbul.

If we calculate in elastic condition we see:

a) The maximum deflections are, in this case, of 4.5 mm.
b) The maximum moments are of 0.32 Ton × m in a punctual case that is rested by the towers, that have not been considered in the calculation in the practice. In general, in significant areas the maximum moments are of 50% of this value.

The Fig. 8 shows deflections, Bending Moments and Principal stresses.

c) The maximum tensions are, in general, smaller than 13 ton/m^2, although, locally, in areas that in fact go reinforced and that we have not considered they arrive to 82 Ton/m^2.

This cover, of great size and form, misses it is extraordinarily effective whenever they stay its reinforcement nerves.

Church of pilgrimage of Steinhausen. Nicholas Zimmerman

We are before a seemingly very simple form, of extraordinarily lengthened elliptic plant, because it

Figure 7
Interior of the Sanctuary of Kappel

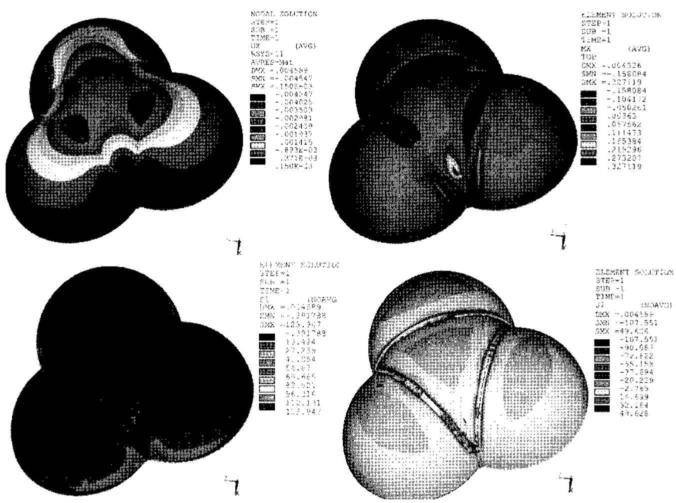


Figure 8
Deflections, Bending Moments and Principal Stresses obtained by the Finite Element Method

keeps a relationship from 5 to 3, what accentuates the directionality of an initially centralized plant (Fig. 5b). The dense ornamentation of the high part camouflages the form that becomes difficult to interpret in its simplicity. To it collaborates it the target of the supports that leaves seemingly suspended this cover. Here it is more difficult to find precedent, since the elliptic plants built so far are supported in all their contour. This and the little thickness of the vault, makes it an unbeatable novelty until Neuman enlarged the range of solutions.

The used geometric pattern is an ellipsoid upon ten supports (Fig. 5b). If we calculate in elastic state, we obtain the following conclusions:

a) The maximum deflections are of 0.226 mm. to own weight .
b) As for the maximum moments, except for due punctual areas to the grid of the mathematical pattern, the maxima are of 0.13 Tonxm. in the area among cylindrical openings or of 0.3 Tonxm. in the same perforations reinforcement. This is false in our case since they have areas specially reinforced by ornamental elements (Fig. 9).
c) As for the tension stresses, you leave in the Figs. 10 that the 9 Ton/m2 doesn't surpassed in any point.

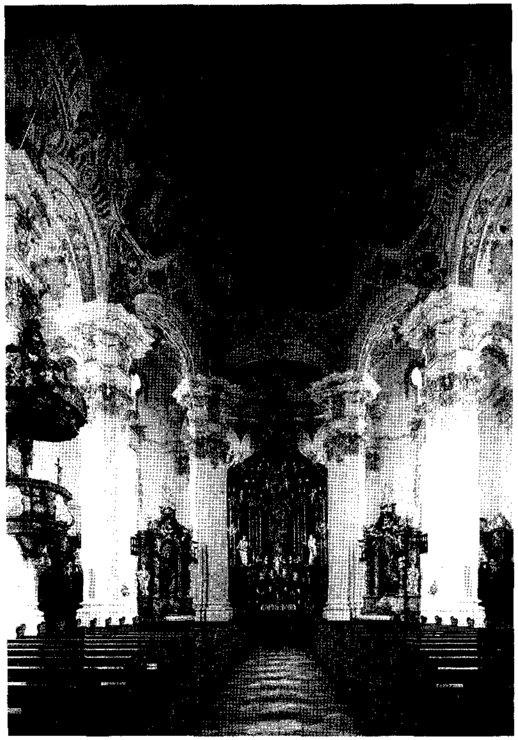


Figure 9

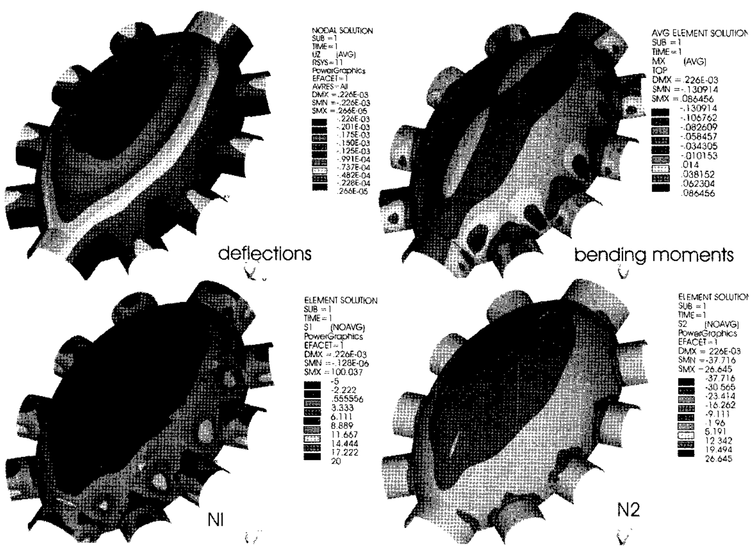


Figure 10
Deflections, Bending Moments and principal Stresses

On the whole it means it that it stops own weight and static loads give a safe state. This structure cannot have pathology and its design is good.

The Fig. 10 shows Deflections, Bending Moments and Principal Stresses obtained by the Finite Element Method.

FRANCISCAN CHURCH OF VIERZHENHEILING. BY BALTHASAR NEUMAN

Neuman was able to increase the complexity in the ways, even with more dimensions, by means of intersections of simple sheets fundamentally of type ellipsoid. We analyze two of their works although this is a to say in such a fruitful author that he made of each proposal a new investigation. In the case of Vierzhenheiling the chained convex spaces are looked for and they reveal the big geometry knowledge that had the contemporary architects, and the space domain that allowed them to it to have.

The geometry is extremely complex and the vault is formed by the intersection of three ellipsoids, two spheres, two cylinders and twenty-two cylindrical openings, in such a way that the whole group supports on twenty-four points (Fig. 6a). All the intersections

Figure 11
Internal view of the shell

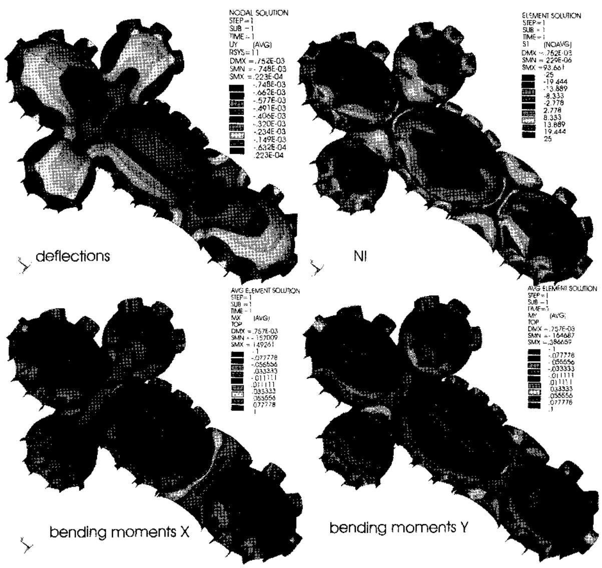


Figure 12
Deflections, Bending Moments and Principal Stresses ob tained by the Finite Element Method

and borders are considered finished off by nerves of 0.5×0.5 m. (Fig. 11). The analysis reveals behaviour as a shell with very good results that approaches to the membrane state, with the exception of some points.

a) In the Fig. 12 we appreciate the displacements that, in the worst in the cases, they don't overcome the 0.7 mm. in the key of the intersection cylinders.
b) As for the tension stresses we have a widespread state that it oscillates between 3 and –3 Ton/m^2. In the intersection nerves we even have 30 Ton/m^2 of traction stresses. For compression stresses, in the kidneys of the domes, it arrives to 30 Ton/m^2.
c) For the bending moments they oscillate among +/–0.15 Tonxm in the X direction. Something greatest are in the nerves in the Y direction. although inside the acceptable range.

Benedictine church of Neresheim. By Baltasar Neuman

The Church of Neresheim is still more complicated since, in spite of the symmetry appearance regarding the central dome, this is not such and the front part and the back one are different. On the other hand they are not cylindrical the agreement surfaces among each two domes but rather they are also ellipsoids.

The architecture of this church is extremely complex and it is one in the most complicated masterpiece in the whole Baroque (Fig.6b). Here we have the intersection of seven ellipsoids of different size and height with spherical canyons, in accordance with the geometry of the Fig. 13.

The structural behaviour of the group is deduced from the calculation by means of Finite Elements with the following results:

a) In the Fig. 14 are appreciated the displacements that, in the worst in the cases, they don't overcome the 16 mm. in the key of the intersection cylinders.
b) As for the tension stresses we have a widespread state that it oscillates between –9 and 3 Ton/m^2. In the pendentifs we even have 60 Ton/m^2 of traction stresses.
c) For the bending moments they oscillate among +/–0.20 Tonxm in the X direction. Something greatest are in the nerves in the Y direction although inside the acceptable range.

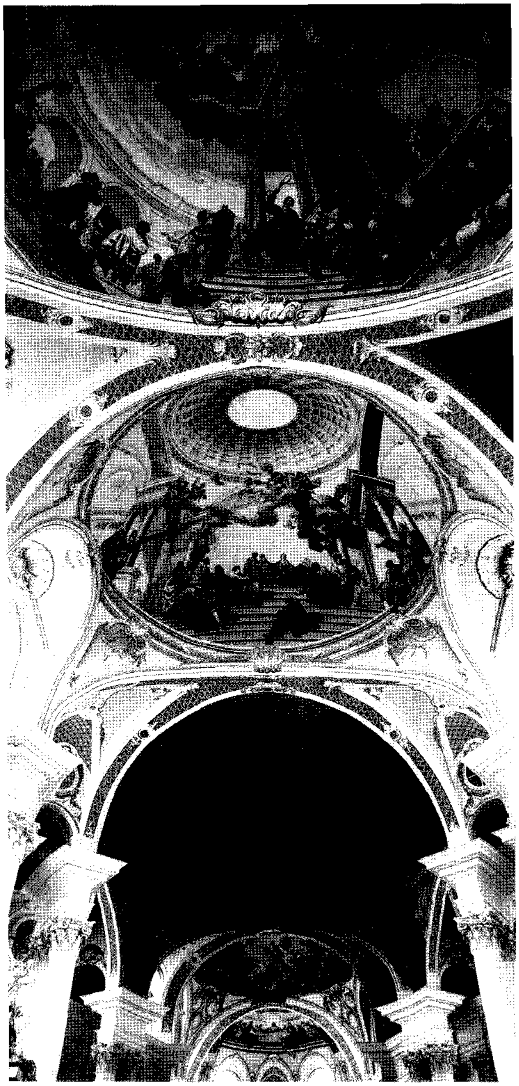


Figure 13
Internal view of the domes

Conclusions

Although it is difficult to establish some conclusions at the present state of the research we want point on

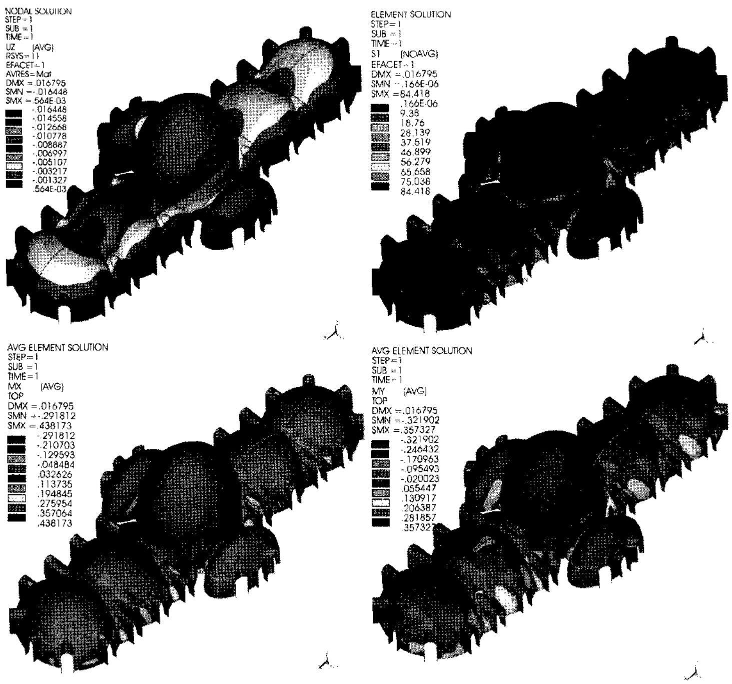


Figure 14
Deflections, Bending Moments and Principal Stresses obtained by the Finite Element Method

the interest of this kind of construction so little studied and so interesting. Our interest is to extend the study to all the Baroque proposals that are so differentiated of other styles, developed before or after.

A first approach allows to settle down some previous conclusions:

- The complexity of the layout is not a capricious artifice, since structurally they are of great effectiveness. Any other form would give bigger stresses and it would require other thickness.
- The behaviour like membrane is demonstrated and the flexions are not significant.
- We can say that they are the structures more complicated shells never built, and it will locate it, for own merit, the Centre European architects among the most important designers in structures of the whole History.
- The membrane state is accentuated in the borders in that some reinforcement nerves pick up the non balanced efforts.
- Another not explored chapter is that of the used constructive systems, since they are not geometries that admit layouts to line neither even wooden guidelines. They very probably never used complete scaffolding just as today in day we make with the armed concrete. When we can consent physically to the back of the vaults we will be able to advance some proposal.

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