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The «Torre Grossa» in San Gimignano: Experimental and numerical analysis

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The «Torre Grossa» (literally *big tower*) is the most relevant within the 13 towers which, nowadays, characterize the skyline of San Gimignano, the famous small town close to Siena.

An extensive study on the tower has been performed within the framework of the research contract called «San Gimignano Project». Research involved local Authorities (Municipality of San Gimignano, Tuscany Regional Administration) and four Departments of different Tuscany Universities (see Bartoli and Mennucci 2000). Even if all the towers have been studied in past years under several different points of view (mainly for architectural and archaeological investigation), no studies were performed in order to check the «safety» of the monument under a structural point of view.

Historical masonry buildings exhibiting a prevailing vertical character, such as towers and bell-towers, represent a structural typology with several common aspects: they are slender tall structures, which mainly have to support their own weight. These characteristics, together with all damages induced by several different factors during the years, make them particularly vulnerable with respect to (even small) base movements, such as those provoked by seismic actions or base settlements; the crack pattern which is inevitably present on these structures appear more or less typical of this kind of monuments.

Moreover, the evaluation of structural reliability of towers and similar structures is quite demanding: these structures possess a low safety margin with respect to external actions, because of the high level of stress induced by the self weight if compared with the ultimate resistance of the materials utilized in the construction. The masonry characteristics and the compression level are besides responsible for the very low ductile overall behaviour of the whole construction. In the past years, several examples of sudden collapse of important towers have been experienced: in 1989 the Torre Civica in Pavia felt down (Macchi 1993; Binda et al. 1992), while in 1993 a collapse interested the bell-tower of St. Magdalena Church in Goch. The collapse of the San Marco bell tower in Venice back in 1902 is another worldwide known example of a sudden tower failure.

The Torre Grossa in San Gimignano (Figure 1) is a 60 m height masonry tower, which presents a square cross section with a side of about 10 m; the masonry is constituted by two facing walls with filling material: the external wall is composed by 20–30 cm thick stone masonry made by prismatic blocks and minimal mortar joints; the internal wall is made by brick masonry with a thickness of about 25 cm; the core, with unknown mechanical properties, have a quite good cohesion (where it has been observed). The overall thickness of tower walls is about 2 m. The tower, built during XIII century, is located in Piazza Duomo (Square of the Cathedral), adjacent to the Palazzo Comunale (Town Hall) and the Cathedral.

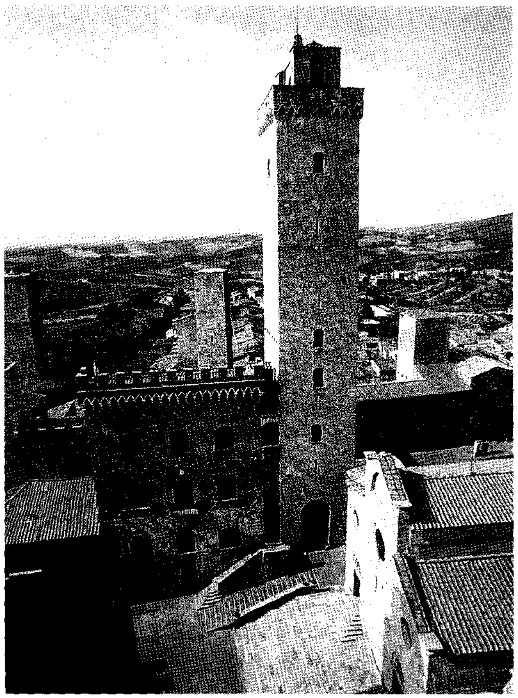

Figure 1
The Torre Grossa in San Gimignano

The aim of the research work was to assess the structural safety of the tower, with respect to seismic actions mainly; at the same time, the goal of the research project was the tuning of a diagnosis procedure, which could be used also in the analysis of the other towers.

MULTIPLE LEAF MASONRY

The type of masonry constituting the tower (made up by three different layers: the external stone masonry, the in-fill material and the internal brick masonry) is often referred as a «multiple leaf» one: the overall mechanical behaviour is quite complex and it is not easily described by very simple models. As a matter of fact, the behaviour of the filling material (constituting the thicker layer) can be intermediate between two limit cases (see e.g. Egermann 1996).

In the first one, the whole structure could be considered as a «multi-layered material». Supposing that the core is composed by a material with sufficient binding, vertical actions can be distributed among the layers according to Hooke's law, then the bearing capacity depends on the relative stiffness of the layer themselves. In addiction to vertical stresses, horizontal stresses can arise in the two outer skins because of the lateral deformation of the in-fill due to the Poisson's effect.

In the second one, the in-fill can be treated as a material possessing no cohesive properties, due to the little or no binder content. In this case, vertical loads are supported only by the external skins, and horizontal loads arise at the interface layers between the core and the outer masonry, approaching the same limiting distribution typical of a «silo». Vertical loads are transferred from the core to the skins by the friction between materials, producing an increment in the vertical load to be supported by the external layers.

The real behaviour of a multiple leaf masonry is intermediate between the two limit cases; as a matter of fact the actual binding of the core material is neither absent nor stiff enough to ensure a full

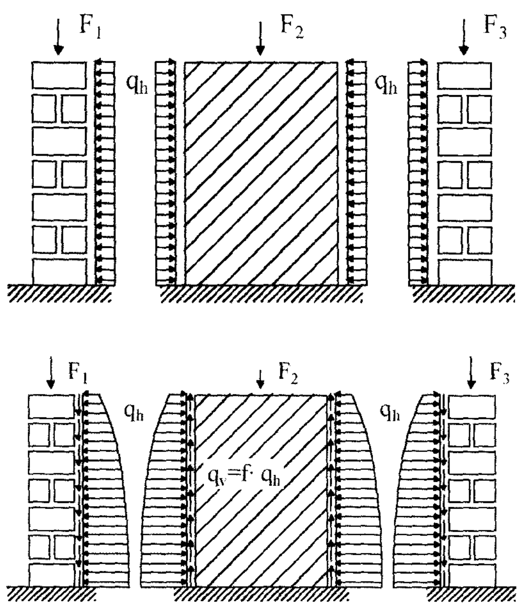

Figure 2
Multiple leaf masonry-Left: the masonry as a «multi-layered material»; Right: the masonry modelled as a «silo» containing a material with low cohesive properties

transmission of vertical loads. Moreover, a certain connection exists between the two layers: this can be either constituted by some brick layers or by horizontal tie-bars, so that ad additional mechanism could arise. Inside the in-fill, several «arch mechanisms» can be thought to be present, where horizontal ties are constituted by the connections between the two external layers, then reducing the horizontal action on the outer skins (see Figure 3).

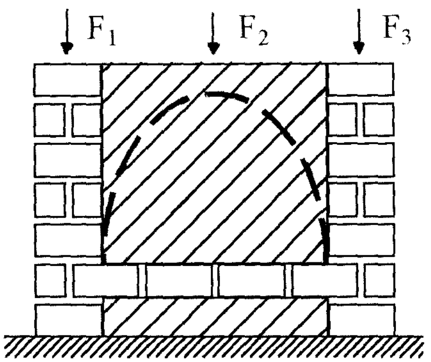

Figure 3
The «arch mechanism» inside a multiple leaf masonry

In a first step of the study, several numerical models have been set up in order to evaluate the stress state of the masonry under different loads. Then, an extensive experimental campaign has been performed in 1996, so as to better understand the actual behaviour of the monument. Differences between obtained results and those from the numerical model (especially for the stress state in the external stone masonry) required further experimental tests (performed in late 1999), mainly devoted to flat-jack tests on stone masonry as well as to get a clearer insight into the stone characteristics and the crack pattern affecting the tower.

Experimental tests

Experimental tests on the Torre Grossa consisted both in «in situ» tests (in order to identify the global structural behaviour and the local masonry characteristics) and in laboratory tests (to single out the mechanical characteristics of the tower constituting materials).

In the «in situ» experimental campaign both dynamical and mechanical tests have been performed.

Dynamical tests have been performed to evaluate the effects induced on the structure by some external actions, such as wind loading and earthquakes. Tests have been executed by using a vibrodyne placed at the top level of the tower, and recording the structural response by means of some seismic accelerometers and velocity transducers. The analysis of recorded signals allowed the identification of the main dynamic characteristics (natural frequencies and eigenmodes). Effects induced by bell movements have been recorded too (on the top of the tower one big bell is located, and its movement is strongly reflected on the structure).

Mechanical tests involved flat jack tests at several locations, so as to determine the local stress state as well as the mechanical characteristics of the masonry (mainly the Young' modulus by «double» flat jack tests); some samples have been also collected to individuate the filling characteristics.

Laboratory tests consisted in crushing tests on stone samples, in order to estimate the ultimate strength of the stone as well as its mechanical characteristics. 37 different samples have been tested, the ultimate strength varying from 43.35 MPa to 65.21 MPa.

All the obtained results have been used to define a numerical model, which was able to reproduce, as close as possible, the actual behaviour of the tower. The model has been built starting from results obtained from the geometrical and architectural survey of the monument, followed by a phase in which mechanical characteristics obtained from experimental tests have been assigned to the materials constituting the model. In the final step, the restraints of the tower have been identified, especially for the part of the tower adjacent to other buildings; the correct restrain level has been accepted when both main natural frequencies and eigenmodes in the model were correspondent to the recorded ones.

Flat jack tests

As it is well known, the flat jack test technique for the determination of both local stress state and stiffness properties in a masonry panel is based on the tensional release induced by horizontal cuts performed on masonry walls. The test has been introduced starting about from 1979 and it is now widely employed. Main steps of the procedure are the following:

- placement of measurement bases;
- execution of the cut by means of a circular saw (usually within a mortar bed joint); the subtraction of resistant material causes a partial closure of the cut, inducing displacements accounted by the measurement bases;
- insertion of the flat jack into the cut;
- increasing of the oil pressure inside the jack until the original displacement state of the masonry measured before the cutting is recovered; in such conditions, the action of the flat jack reproduces the one previously given by the material removed by means of the cut; some interpretative parameter are defined in order to achieve the stress state in the masonry basing on the oil pressure inside the jack.

The stress acting before the cut is usually obtained following the expression:

$$\sigma = S_f = K_m \cdot K_a \cdot P_f \qquad (1)$$

being σ the stress acting before the cut, S_f the action of the jack, K_m a jack parameter depending on jack characteristics (geometry, stiffness, construction), K_a a second parameter depending on the ratio between the cut and the flat jack surfaces and P_f the jack internal oil pressure.

Distances between measurement bases are recorded initially, after the execution of the cut and following each increment of the pressure of the jack. Three or more measurement bases are usually employed (Figure 4), following the recommendations reported by the American Society for Testing and Materials (1991).

As mentioned above, the Torre Grossa is built by two facing walls with filling material and several pinning: due to the remarkable stiffness change among the three layers which compose the tower, the stress state induced by dead weight results to be sustained mainly by the stone external wall, which results to be subjected to a strong compression stress condition.

The main part of flat jack investigation have thereby been performed on the external stone masonry wall, in order of evaluating the actual compression stress state and, employing a couple of jacks, the overall Young modulus. Table 1 summarises some of the main obtained results by flat jack tests.

Table 1. Flat jack tests results

TEST # (year)	MASONRY TYPE (test type)	STRESS VALUES [MPa]	YOUNG' MODULUS [MPa]
1 (1996)	BRICK (double)	1.594	6130
2 (1996)	BRICK (double)	0.922	2270
3 (1996)	BRICK (single)	1.555	—
4 (1996)	STONE (double)	4.416	11530
5 (1996)	STONE (single)	4.416	—
6 (1999)	STONE (single)	4.800	—
6A (1999)	STONE (double)	4.800	11350
7 (1999)	BRICK (single)	0.864	—
8 (1999)	STONE (single)	5.568	—
9 (1999)	STONE (single)	6.720	—

In the examined case, the use of Eqn. (1) gave stress values in strong discordance with those from numerical analyses. In a second experimental phase, flat jack tests have been repeated, also evaluating the influence of their positions: in mortar bed joints or within a single stone block. In every case, a material behavior strongly different from the linear elastic one initially assumed was observed.

In order to achieve a better interpretation of masonry behavior, a particular test has been performed within a single stone block, so as to minimize effects from rigid rotations of other blocks or from the irregularities of joints surfaces (Figure 4). The placement of many measurement bases along the cut and the small pressure increments imposed during the load phase allowed an accurate description of the test, permitting a reliable interpretation of data. Figure 5 shows displacement curves obtained for the five measurement bases vs. the flat jack internal pressure.

The analysis of the obtained results reveal several relevant aspects concerning the behavior of masonry during the test:

- the initial displaced configuration and the following ones (correspondent to each pressure increment) result to be almost symmetric and much more regular with respect to the ones obtained from tests performed along bed joints;
- the incremental displaced configurations (i.e. the ones measured with respect to the configuration subsequent to the cutting) results to be represented with good reliability by two straight lines, showing thereby a very different behavior with respect to a linear elastic scheme (in this case, in fact, incremental displaced configurations would assume the same shape of the global displaced one);
- in correspondence of the recover pressure for the central base (no. 3 in Figure 5- about 56 bar), the extreme bases no. 1 and 5 present a recover of about 65% of the initial displacement: it is thereby evident the occurrence of localized (both elastic and inelastic) displacements along the cut boundaries.

To evaluate the results it is then necessary to abandon the linear elastic behavior scheme. A suitable model has been proposed by Bartoli, Chiostrini e Innocenti (2000) in order to take into account the nonlinear masonry behavior. The evolution of the strain state has been modelled by introducing an «equivalent» beam that, according to the crack evolution, possesses variable stiffness properties as a function of the applied load.

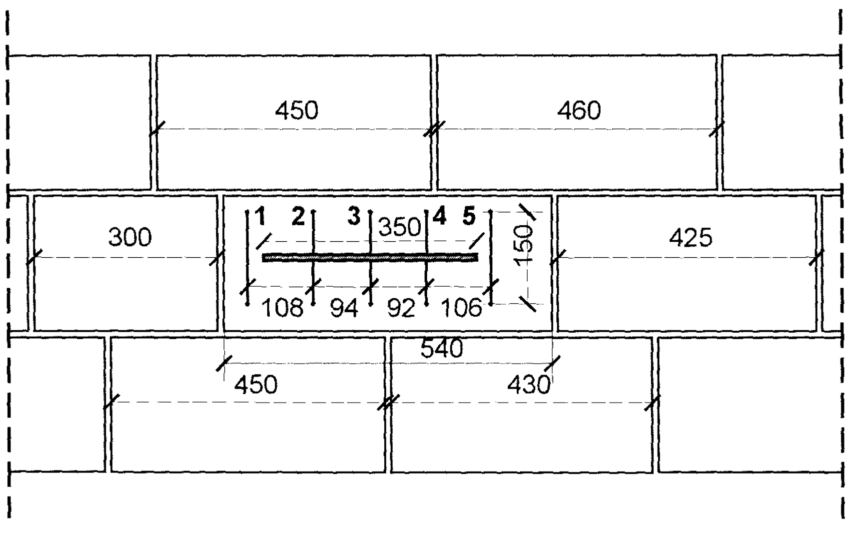

Figure 4
Base positioning [mm]

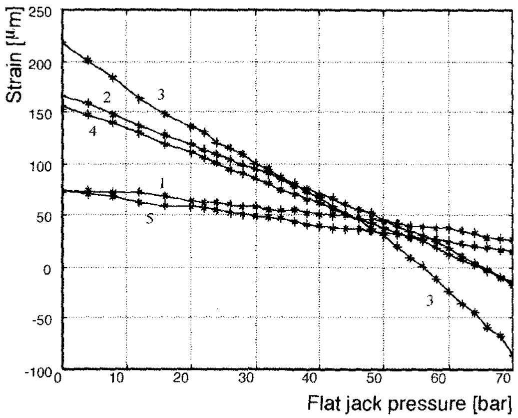

Figure 5
Strains vs. flat jack pressure for all measurement bases

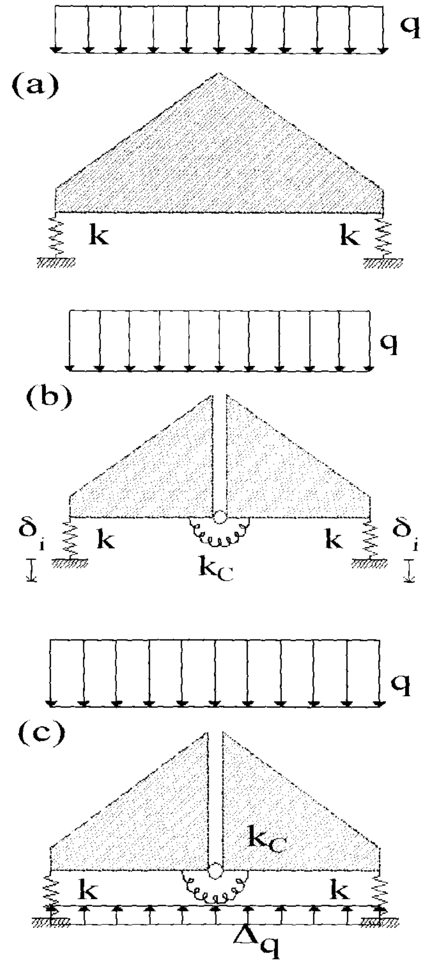

Figure 6
Equivalent beam evolution during the flat jack test

The beam (which has a trapezoidal shape) is elastically supported by the adjacent masonry; when a certain value of the stress has been exceeded in the central section of the beam (during the cutting procedure) the beam itself evolves to a different one, where an elastic hinge is located just at midspan, accounting for the stiffness reduction.

The condition of the beam described above (initially linear elastic, then formation of the hinge at midspan and inelastic displacements of supports) corresponds to the situation of the masonry sharply achieved with the execution of the cut. Effects of the load phase induced by the flat jack is finally represented by the upward load in Figure 6c.

Figure 7 depicts the load cycle of the equivalent beam in which a), b) and δ_i segments correspond to the cutting, while the c) phase represents the loading path performed by the flat jack.

Parameters of the equivalent beam defined above are: the material Young modulus E, the elastic stiffness of the end supports k, the elastic stiffness of the midspan pin k_C, the inelastic displacement of the end supports δ_i, q^* (formation of the middle span hinge) and q_{max} (actual vertical stress); all these values have to be estimated through experimental results. The main guidelines of the procedure are the following:

- k_C is determined from the incremental displaced configuration as the ratio between the bending moment at midspan of the beam and the relative angle between the two segments which form the configuration itself;
- k is determined referring to the numerical model of Figure 6a;
- E, q^*, q_{max} and δ_i are not independent and have to be calculated imposing the equivalence of experimental displacements from the flat jack test and the deformed configuration of the equivalent beam.

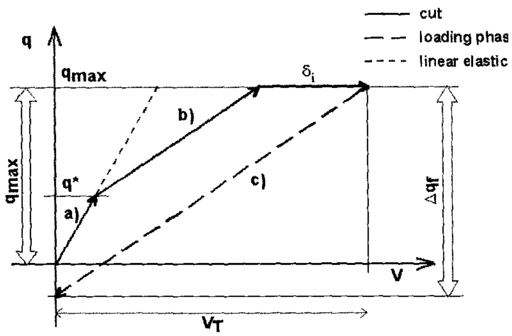

Figure 7
Load-displacement curve for the equivalent beam

The procedure has been utilized to process data from different flat jack tests, obtaining q_{max} values result substantially lower than $S_f = K_a \cdot K_m \cdot P_f$ from Eq. (1), confirming in this way that the cited formula leads to an overestimation of the actual stress state of masonry.

An additional «correction coefficient» β could then be introduced, accounting for the possible nonlinear behavior recorded during test

$$\sigma_{actual} = \beta \cdot S_f = \beta \cdot K_m \cdot P_f \tag{2}$$

Referring to Table 2, it is possible to conclude that in the case of the Torre Grossa in San Gimignano, the application of Eqn. (1) leads to an overestimation of about 20% with respect to the actual stress value in masonry walls.

Table 2. Comparison between results from Eqn. (1) and those from the proposed model

Test	Stress from Eqn. (1) $S_f = K_a \cdot K_m \cdot P_f$ [MPa]	Identified stress σ_{actual} [MPa]	$\beta = \frac{\sigma_{actual}}{K_a \cdot K_m \cdot P_f}$
Within a stone	6.72	5.50	0.81
Within a mortar joint	4.80	3.80	0.80

Determination of an «equivalent» stone

Three different stone types have been used to build the external wall of the Torre Grossa: a porous limestone, the travertine and a kind of limestone called «amphystegina». Travertine can be further subdivided into three different classes according to its porous' degree; in the following the three levels has been named as low porous, medium porous and high porous.

By laboratory tests, mechanical characteristics of all the different types of stone have been determined; Table 3 reports the obtained values.

Table 3. Mechanical characteristics of different stone types

Stone types	Young' modulus E [MPa]	Characteristic strength [MPa]
Porous limestone	45000	36.33
Travertine #1 (low porous)	35000	31.06
Travertine #2 (med. Porous)	33000	21.83
Travertine #3 (high porous)	12500	6.74
Limestone «amphystegina»	48000	32.66

Some researchers of the University of Siena have then proceeded to a localization of all the different types of stones on the North and South walls of the tower, so producing an accurate mapping of each single block; from the obtained results, it has been observed that travertine, the weakest among the used stone types, is predominant with respect to limestone, that limestone has been used in very localized part of the walls, and that, on average, the quality of the used stones is quite good (Dipartimento di Archeologia . . . 1997).

Starting from the obtained mapping, numerical models of some representative masonry panels have been set up. Each «numerical» panel has been modelled by using *Solid65* elements within those available in the finite elements code Ansys (Swanson Analysis System, Inc. 1996); this kind of element has been chosen because of the possibility of performing both cracking and plastic analyses. Each panel have been restrained so as to reproduce a compressive laboratory test: nodes on the lower part have been fixed, while side nodes have been restrained in order to avoid any displacement in the transversal direction with respect to the applied vertical load. Upper level nodes have been restrained horizontally, while test has been simulated by imposing progressive vertical displacement and evaluating the force level within the panel; displacement increments have been maintained very low (0.5 mm for each loading steps) so as to perform an accurate step-by-step nonlinear analysis.

The aim of the numerical simulation was to individuate, according to different stone arrangement within the panel, several load-displacement curves, in order to define an «equivalent» stone, which could be used in the modelling of the whole structure.

Plastic behaviour of the materials has been described by means of the Drucker-Prager criterion; all the necessary parameters (cohesion, c, friction angle, ϕ) have been defined starting from uni-axial tensile strength f_t and compressive strength f_c.

Cracking of the materials have been introduced by means of the Willam-Warnke model (1974), where parameters have been tuned starting from uni-axial result.

All the introduced parameters have been then modified in order to take into account the actual materials' behaviour: as a matter of fact, the real behaviour is intermediate between a «purely frictional» one (characterized by a null value of the dilatancy, δ) and a «purely dilatative» one ($\delta = \phi$). Parameters' modifications are also necessary for taking into account the dissipative phenomena arising when cracking occurs; cracking, if no modifications are made, is in fact taken into account as a stiffness reduction only (Davis 1968; Chen 1975).

By means of the mapping of the South wall of the tower, ten different areas in which masonry was composed by travertine only have been singled out; the areas have been chosen so as to obtain an exhaustive description of all the main possible block combination in the stone masonry of the Torre Grossa (Figure 8). Each single area has then been numerically reproduced, determining its ultimate strengths; ultimate strengths have been thought as dependent on the percentage of different travertine types present in the panel as well as on their reciprocal position within the panel (Bartoli, Casamaggi and Spinelli 2000).

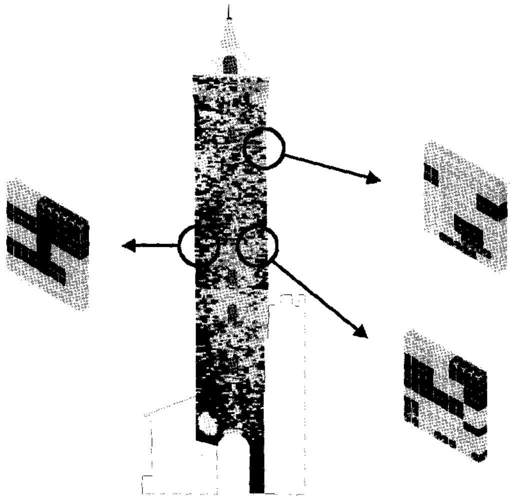

Figure 8
Mapping of travertine' stones and panels' numerical modelling (South side)

Once the ultimate strength has been determined for each panel, a suitable relationship has been found which allowed estimating the compressive strength of an «equivalent» stone able to represent the behaviour of the masonry, which constitutes the external wall of the tower.

A parameter R has been introduced, taking into account the percentage of the three different types of travertine within the panel. The parameter has been assumed as

$$R = a_1 R_1 + a_2 R_2 + a_3 R_3 \tag{3}$$

where R_1, R_2, R_3 are the ultimate strengths of low porous, medium porous and high porous travertine respectively, while a_1, a_2 and a_3 represent the respective percentages.

Once R values have been evaluated for each single panel, the same parameters has been evaluated for the whole tower (the tower being constituted by a 37% of low porous travertine, a 38% of medium porous travertine and a 15% of high porous travertine) and it has been called R_{tower}; R values have been plotted vs. the actual ultimate strength of each panel and some best fit approximation curves have been evaluated. The ultimate strength for the whole tower has then been evaluated by using these approximation curves; the average between all the values obtained by intersecting the approximation curves with the R_{tower} value has then resulted to be equal to 9.31 MPa. The «equivalent» stone constituting the tower can therefore supposed to possess this value as ultimate strength and this one can be used as a reliable value for characterizing the materials used in the numerical model of the whole structure.

THE SURVEY OF THE CRACK PATTERN ON THE TORRE GROSSA

The investigation on the Torre Grossa allowed to point out the crack pattern on the external walls of the monument: the survey was possible thanks to the presence of a special moving scaffolding fixed on the North and South walls, which allowed to investigate the masonry at a very close distance. The two walls are affected by a complex system of cracks, while fewer cracks are also present in the other two external walls and in the internal brick masonry.

The procedure consisted in fixing on the external surface some measuring tapes in order to have a constant reference during all following investigations; as many pictures as those necessary to cover the whole surface of the tower have been taken from a distance of approximately one meter. The global surface has then been reproduced by joining adjacent pictures, so to have a complete reproduction of all the cracks; from the whole picture set, the full crack pattern on both the two investigated sides has finally been singled out (Figure 9).

The crack layout is in a good agreement with the one usually characterising masonry towers: main cracks are in the vertical direction, i.e. the same direction of main compressive stresses. The path followed by each single crack is obviously influenced by local masonry characteristics, such as mortar joints, which constitute preferred way for the cracks' development, even if, in several points, cracks pass through the stone blocks.

On the South side of the tower, a big fracture has been observed, and it is present in the internal brick masonry too; such a crack, presenting a maximum amplitude of about 1 cm, begins at the level of the upper window and goes down as far as the level of about +20 m with respect to Piazza del Duomo level.

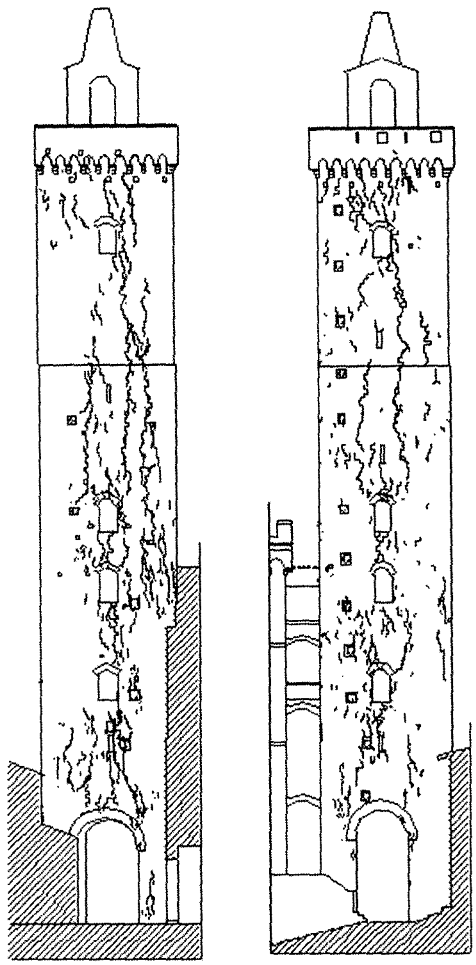

Figure 9
Crack pattern on South and North sides of the Torre Grossa

The crack is on the right part of the wall and, in its lowest part, splits into several minor cracks, ending in correspondence of the Palazzo Comunale, the historical building adjacent to the East side of tower. The presence of several cracks in its lowest part is maybe due a local stone crushing caused by the high compressive stresses.

On the North side, fewer cracks have been singled out, even if a larger diffuse damaging is present, caused by its orientation; in several points, joints appear opened, depending on the breaking up of the mortar. Moreover, on this side a wide number of localized damaging is due to grenade shots, which hit the tower during Second World War. The two main fractures are represented by a wide crack on the central-right side from a level of about 25 m up to a level of 43 m, and by the one close to the left side edge of the tower. Along the tower corners, material has been expelled at several different heights.

From the surveyed crack pattern, two main aspects arise: the first is represented by the fact that base settlements are not responsible for the crack layout, because of the absence of cracks in the lower part of the tower; the second aspect is related to the fact that self weight cannot be the only cause of the cracks' occurrence, given that fractures are also present in the upper part of the tower, just below the small arches under the tower's battlement.

An interpretation on the cause of the main crack presence

As a last part of the research work, a study has been performed on the possible cause of the main crack on the South side of the tower. Several analyses have been performed on a numerical model of the whole tower, looking for high vertical (compressive) and horizontal (tensile) stress levels.

In the numerical model, the thickness of the two masonry layers has been maintained as constant along the tower height; the stone masonry thickness has been assumed equal to 20 cm, while the internal brick masonry presents a thickness of 25 cm; the filling thickness has then been varied along the tower's height, according to the reduction of wall thickness. Main openings have been introduced in the model, together with all the cavities pointed out during the architectural survey; the consequences of the collapse of a wide part in the South-West edge due to a lightning which hit the tower in 1632 (the collapsed part was re-built in 1650) has been investigated in order to give an explanation to the cracks' presence.

The part collapsed after the lightning event has then been removed from the model; the stress pattern on the tower due to self-weight only appears substantially different with respect to the one when the collapsed part was still present, and vertical compressive stresses more or less follow the main crack layout. Stress levels are very high and are increasing toward the lower part of the tower; just below the zone restrained by the Palazzo Comunale, vertical stresses are lower and tend to be shifted toward the left part of the wall, where the survey pointed out a wide presence of small cracks, close to the main arch.

According to the vertical compressive stresses, horizontal tensile stresses arise, which could be responsible for the cracks' opening, so that, even in the North side of the tower, zones where highest compressive stress levels have been pointed out follow the main cracks' direction and location.

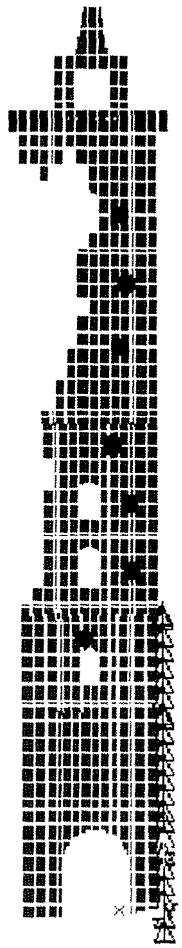

Figure 10
Position of vertical stress resultant at seven different levels (South side)

In order to confirm the hypothesis that lightning could be thought as responsible for the cracks' appearance on the tower, vertical stresses have been investigated by analysing the position of the vertical stress resultant on the South side of the tower at 7 different levels.

The position of the stress resultant at these levels has then been mapped on the side view of the tower, singling out the path followed by the cracks along the tower. As it can be seen from Figure 10, position of stress resultants at different levels matches very closely the shape of the main cracks, then confirming that lightning event played a fundamental role in the cracks' formation. It is quite interesting to note that, under the level of 17 m (which correspond to a zone below the level where the Palazzo Comunale is linked to the tower) the stress resultant is more or less centered with respect to the side of the tower.

It is also to be remarked dynamical effects due to the lightning as well as those derived from the collapse of the masonry should also be considered in the analysis, even if these two phenomena are very difficult to be modelled numerically.

Concluding remarks

In the paper, some of the main aspects related to a research work on the Torre Grossa in San Gimignano have been reported.

Nonlinear behavior of masonry has been observed performing flat jack tests on the wall external surface of the Torre Grossa, in San Gimignano. In order to obtain actual value of stress by means of such tests, an interpretation procedure has been defined, demonstrating that results obtained with the hypothesis of linear elastic behavior of masonry are overestimated of about 20% with respect to actual stress values (confirming otherwise results from numerical simulations obtained by F.E. models).

Starting from some experimental results, a numerical model has been set up, after a preliminary research work on the determination of the characteristics of an «equivalent» stone to be used in the analysis.

An extensive survey has been performed to identify the crack pattern on two of the four sides of the tower: a final numerical modelling confirmed that a lightning that hit the tower in 1632 was probably responsible for the cracks' appearance.

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