Proceedings of the First International Congress on Construction History, Madrid, 20th-24th January 2003, ed. S. Huerta, Madrid: I. Juan de Herrera, SEdHC, ETSAM, A. E. Benvenuto, COAM, F. Dragados, 2003.

The methods of graphical statics and their relation to the structural form

Rolf Gerhardt
Karl-Eugen Kurrer
Gerhard Pichler

In the preface of the 2nd edition of his main work «Die graphische Statik» (1875), Karl Culmann was pleased to report that graphical statics had been taken up widely, but at the same time he complained that it was taught without the mathematical base of the projective geometry. Subsequently, the «Graphostatik» (graphostatics), characterised by its recipe-like structure, became generally accepted.

The graphostatics formed an essential means of intellectual engineering practice. It still referred to the unity of design, calculation and construction. Various examples demonstrate the option of deriving from the graphical examination not only the internal forces required for dimensioning, but also criteria for optimising the structural form. Nevertheless, with the completion of the classic structural theory after 1890, the decline of the graphical statics in favour of analytical calculation methods was already becoming apparent. This corresponded with the desire to rationalise the engineer's work, but it led to the disintegration of the unity of design, calculation and construction.

Today unlike in engineering practice, graphical methods still play an important role in lecturing on structures. The clarity of graphical techniques has a high didactic value, since interdependencies, e.g. between forces and structural geometry, can be directly experienced visually. Architecture and structural engineering students thus are provided with a suitable means of evaluating not least historic structures. Recent examples from engineering practice demonstrate how helpful the know-ledge and use of graphical methods can be for the evaluation and preservation of historic structures: The history of structural methods is therefore an important facet of construction history.

Graphical statics and graphostatics in the work of Culmann

Graphical statics started with the publication of the monograph under the same title between 1864 and 1866. Karl Culmann (1821–1881) had been lecturing graphical statics at Zurich Polytechnic (now known as ETH) since 1860 (Maurer 1998). According to its creator, it is an attempt «to solve engineering tasks that are accessible to geometric treatment with the aid of the new geometry» (Culmann 1864/1866, VI). With «new geometry», Culmann refers to the projective geometry going back to the publication by Jean Victor Poncelets (1788–1867) entitled «Traité des propriétés projectives des figures» (Poncelet 1822), investigating the projective characteristics of figures. How does this projective geometry, which is after all freed from metrics, i.e. from dimensions and numbers, and which is only interested in the relative position of geometric figures, relate to graphical statics, with the latter dealing specifically with the measuring of geometric parameters?

The answer is provided by means of an example, based on Culmann's mathematical proof of the projective relationship between funicular polygon and force polygon, which had already been introduced by Pierre Varignon (1654–1722) (Figure 1).

For the special case of an equilibrium system of forces with one point of application, Culmann found the «structural relation between funicular polygon and force polygon via a planar correlation of the projective geometry» (Scholz 1989, 174). Funicular polygon and force polygon are interchangeable in so far as it does not matter which of the polygons is regarded as the system of the lines of action in addition to the associated funicular polygon. Culmann describes such figures as reciprocal (Figure 2).

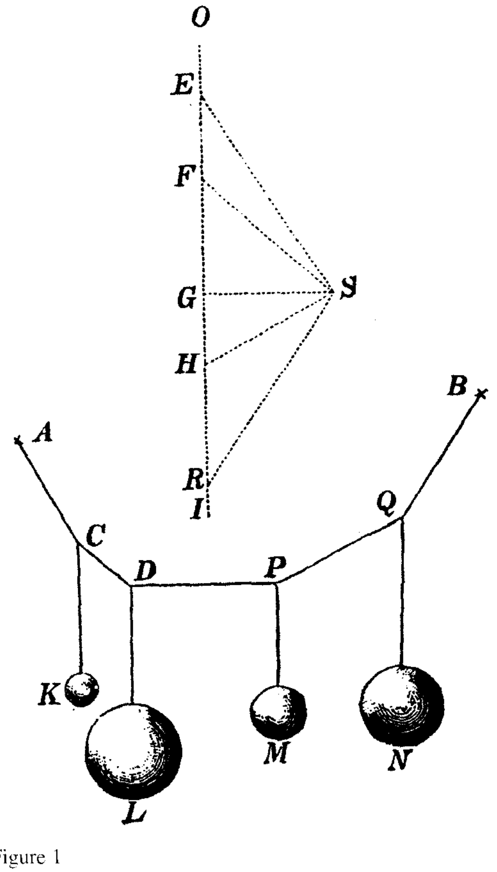


Figure 1
Funicular polygon and force polygon according to Varignon. (Varignon 1725)

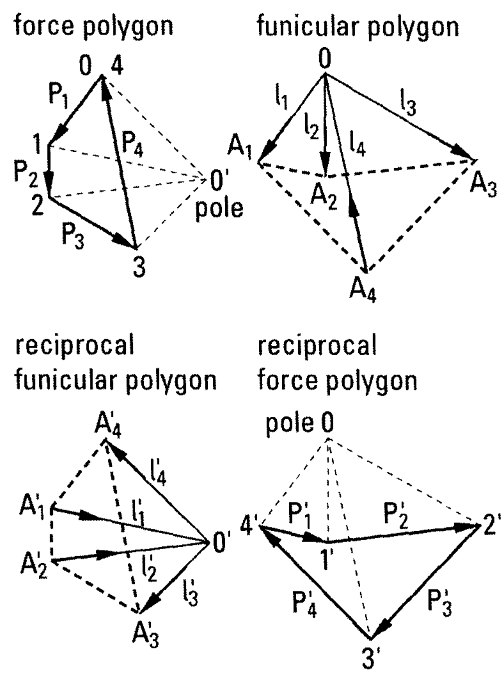


Figure 2
Reciprocal diagrams of funicular polygons and force polygons for plane systems of forces according to Culmann. (Kurrer 2002, 224)

Soon after the publication of Culmann's «Graphische Statik», graphical statics became separated from its applications in the form of graphostatics. Evidence for this can be found in the even more mathematically oriented second edition published in 1875, in which Culmann promises the reader to elaborate on the applications in a second volume (Culmann 1875, XIV). However, this never happened, because Culmann's scientific thinking was too much determined by the ideal of an axiomatic substantiation of graphical statics.

Culmann's visualisation of beam statics via the funicular polygon (Figure 3), which is an important component of graphostatics, is dominated by the operational side that is restrained in the means/purpose relation. Relying on measures and numbers, it rationalises and mechanises an important side of the working process of the engineer through informational consolidation in the form of graphical

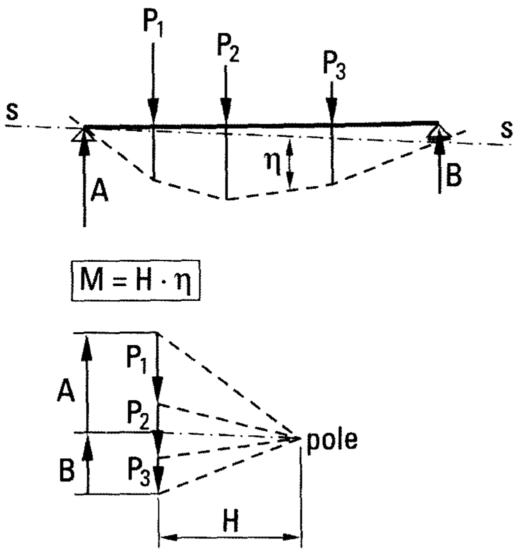
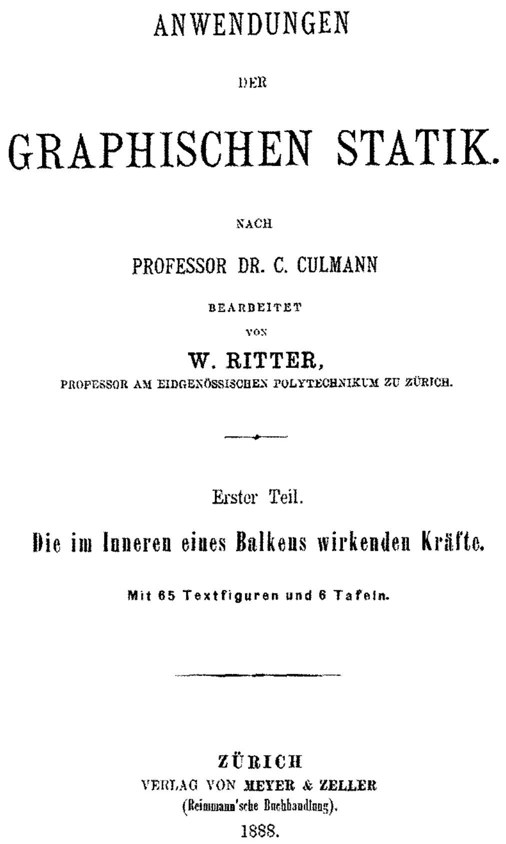


Figure 3
Graphical solution representing the bending moment diagram of a simply supported beam according to Culmann. (Kurrer 2002, 224)



ANWENDUNGEN

DER

GRAPHISCHEN STATIK.

NACH

PROFESSOR DR. C. CULMANN

BEARBEITET

VON

W. RITTER,

PROFESSOR AM EIDGENÖSSISCHEN POLYTECHNIKUM ZU ZÜRICH.

Erster Teil.

Die im Inneren eines Balkens wirkenden Kräfte.

Mit 65 Textfiguren und 6 Tafeln.

ZÜRICH
VERLAG VON MEYER & ZELLER
(Reimmann'sche Buchhandlung).
1888.

Figure 4
Title page of Ritter's «Anwendungen der graphischen Statik». (Ritter 1888)

methods. Graphostatics mediates between the manpower of the engineer and his object, here in the form of ideal static models. Graphostatics represents an important intellectual tool of the engineer, which had its heyday in the last third of the 19th century within the structure of fundamental engineering sciences and structural engineering.

While graphical statics was the keystone of the establishment phase of the structural theory that had started with the trussed framework theory (1850–1875), the emancipation of graphostatics from graphical statics and its cognitive expansion was an important moment in the completion phase of the structural theory (1875–1900). Even Culmann's student and successor at ETH Zurich, Wilhelm Ritter (1847–1906), became disloyal to graphical statics and his own view of its basis on projective geometry, as it was formulated in the foreword of his four-volume work published between 1888 and 1906 «Anwendungen der graphischen Statik. Nach Professor Dr. C. Culmann», Figure 4: He wrote a work on graphostatics, in which this view only appeared in the form of a mere «manner of speaking» (Scholz 1989).

In 1882, Ludwig Tetmajer (1850–1905) recognised the influence of Culmann's graphostatics as follows: «The grand bridge arches that were constructed in Switzerland since 1876 have all been calculated according to Culmann's theory, with contributions from construction enterprises such as Holzmann and Benkieser in Frankfurt, Eiffel in Paris, (. . .) in order to be able to utilise the results of Culmann's research in their construction bureaus.» (quoted after Stüssi 1951, 2).

The renunciation of static calculations, partly caused by the graphostatics, is a historic product of an increasing conditioning of the work of the structural engineer towards static techniques. Nevertheless, during the classic phase of structural theory it offered the opportunity to create fine iron structures. One could even go as far as saying that the graphostatics did not only rationalise the design work of the structural engineer, but at the same time aestheticised it, bearing in mind that the force and construction drawings appear in the dual shape of both the sensory consciousness and the sensory needs (Figure 5). This development reached its peak during the 1880s and 1890s; a prominent example is the Eiffel Tower, which was analysed by Culmann's former student Maurice Koechlin (1856–1946) using graphostatical techniques.

Nevertheless, with the completion of the classic theory of structures after 1890, the decline of the graphical methods in favour of analytical calculation methods was already becoming apparent. This corresponded with the desire to rationalise the engineering process, but it led to the disintegration of the unity of design, calculation and construction.

THE SIGNIFICANCE OF GRAPHICAL METHODS FOR LECTURING ON STRUCTURES

Visual experientiality of interrelations

Whilst in modern engineering practice, graphostatics is largely seen as a historic variety of structural analysis, its techniques are still relevant in lecturing

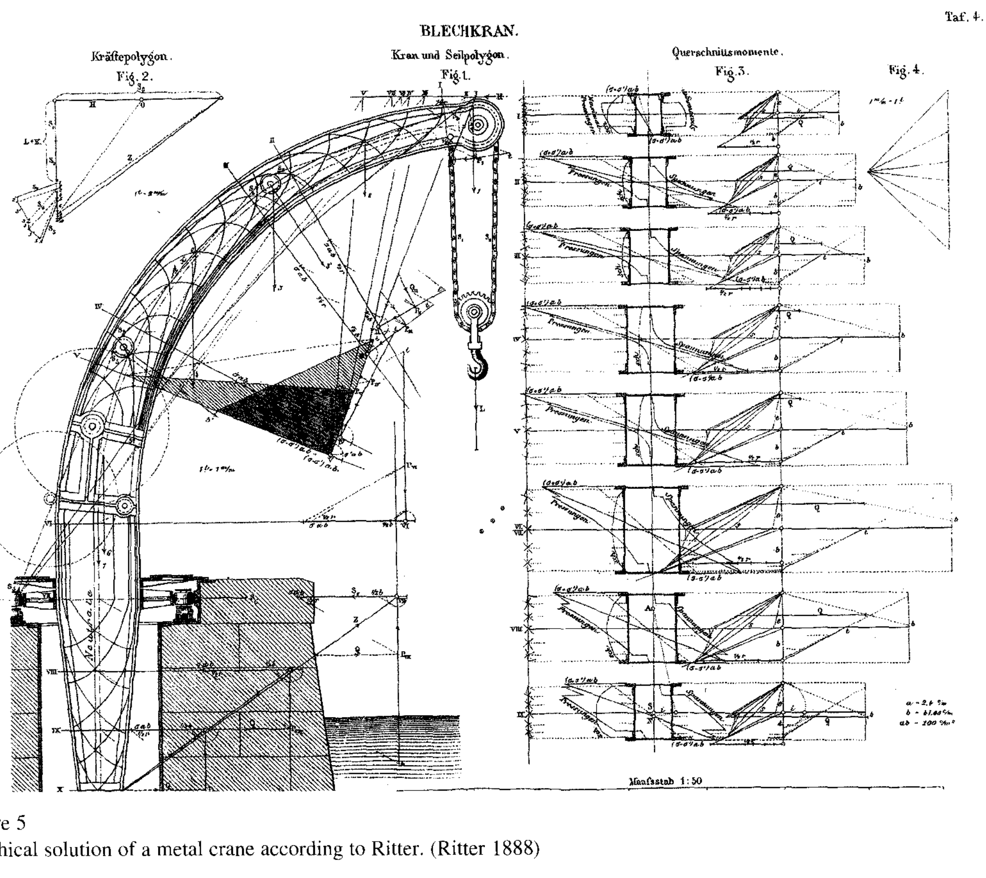


Figure 5
Graphical solution of a metal crane according to Ritter. (Ritter 1888)

on structures. The clarity of the graphical method has a high didactic value, since interrelations and interdependencies, e.g. between structural geometry and forces, can be directly experienced visually.

The Cremona diagram shown in Figure 6, for example, clearly illustrates how a reduction of the forces acting in the members of a framework can be achieved by increasing the depth of the trussed girder under consideration.

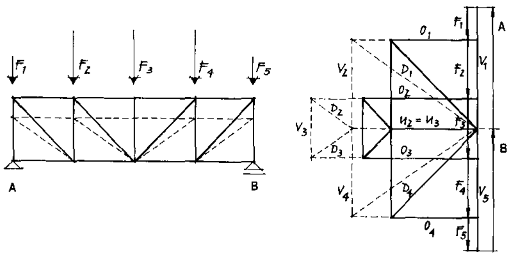


Figure 6
Cremona's diagram of forces for a trussed girder. Comparison of two girders with different depths.

Optimisation of the structural design

Design, dimensioning and construction and their mutual influences can be experienced as a unit in graphical solutions. Criteria for the optimisation of the structural form are almost imposing themselves: The optimisation options are directly apparent and can be implemented directly in the design without the need for complicated calculations.

An example is the three-hinged frame shown in Figure 7. Based on the theory of the thrust line, the supporting forces can easily be determined, and therefore also the bending moments at critical points (corner joints). The graphical optimisation shown in Figure 8 leads to a more balanced distribution of bending moments with significantly reduced maximum values by relocating the hinge. The interrelation between hinge relocation, thrust line and reduction of horizontal reaction forces can clearly be seen in the graphical solution.

For lecturing on the theory of structures it has proven useful to extend the application of graphical

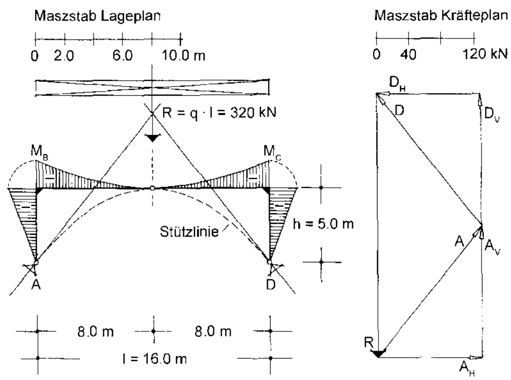


Figure 7
Symmetrical frame with three hinges. Graphical determination of the supporting forces by means of the thrust line. (Gerhardt 2002a, 65)

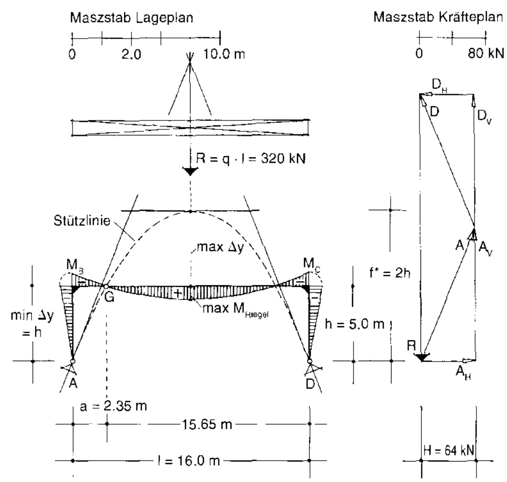


Figure 8
Frame with three hinges. Graphical optimisation of the position of a hinge in order to minimise the bending moments. (Gerhardt 2002a, 69)

solution techniques to statically indeterminate systems, as far as possible with simple procedures. The principle of the thrust line can thus also be used, for example, for examining statically indeterminate frames, Figure 9, by assuming realistic ratios of beam stiffness to post stiffness. The zone in which all 'reasonable' bending moment curves and thrust lines respectively are located can thus be defined quite exactly, despite the statical indeterminacy (Führer, Ingendaaij, Stein [1984] 1995, 201–6; Gerhardt 2002b, 381–3).

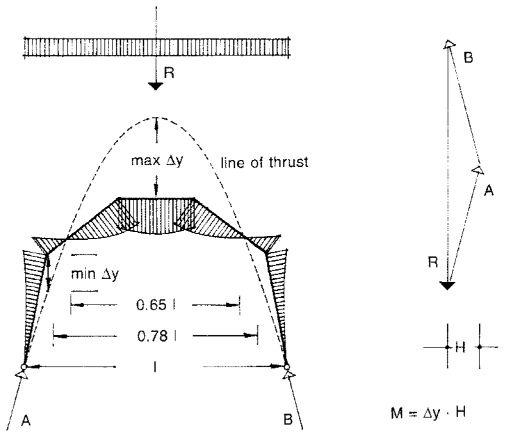


Figure 9
Bending moment diagram of a frame with two hinges. Optimised structural form as it approaches the line of thrust. (Gerhardt 2002b, 383)

«The aim of this procedure is to make the relationship between structural form, external load and the resulting bending moments directly experienceable through experiments with hanging chains.» (Gerhardt 2002a, 8)

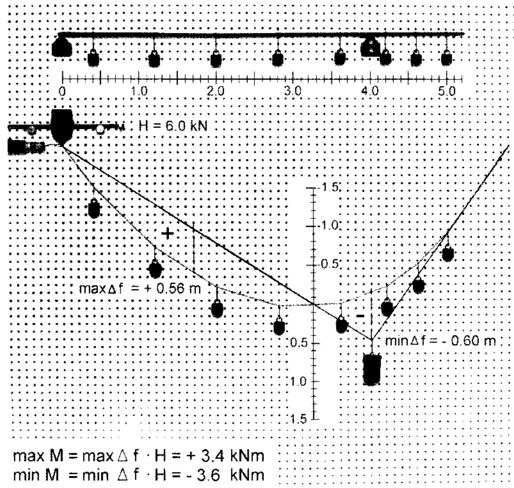


Figure 10
Overhanging beam. Experimentally derived funicular polygon representing the bending moment 'diagram'.(Gerhardt 1998, 35)

The use of demonstration models in lecturing on graphostatics

Experiments with demonstration models have proven didactically helpful for communicating theoretical facts in lecturing on structures. Studies show that theories are easily understood and lastingly memorised by students due to the illustrative quality of model experiments (Faisst 1975).

The graphical method of determining bending moments via force polygon and funicular polygon can also easily be experimentally demonstrated with hanging chains. Here, the behaviour of a hanging chain under load, which resembles the bending moment curve of a beam under the same load, is utilised. The representation of the bending moment diagram via hanging chains as shown in Figure 10 is the materialisation of the graphical funicular polygon.

«For the dimensioning of structural components, the magnitude and distribution of bending moments are a basic criterion and are often crucial. Whilst the internal moments acting in a structural element subject to bending stress are often difficult to imagine, it is much easier to do so for the deformation of a hanging chain under an equivalent load. Via the analogy of the catenary, the qualitative shape of the bending moment curve is thus directly comprehensible. The method of experimental representation of bending moments can thus contribute to the safe estimation of bending stresses independent of calculations, and to the optimisation of load-bearing structures even during the design phase.» (Gerhardt 2002a, 8)

Distribution of forces and structural form

The correspondence of structural form and physical stress inevitably implies both structural and formal aesthetic qualities (Figure 11).

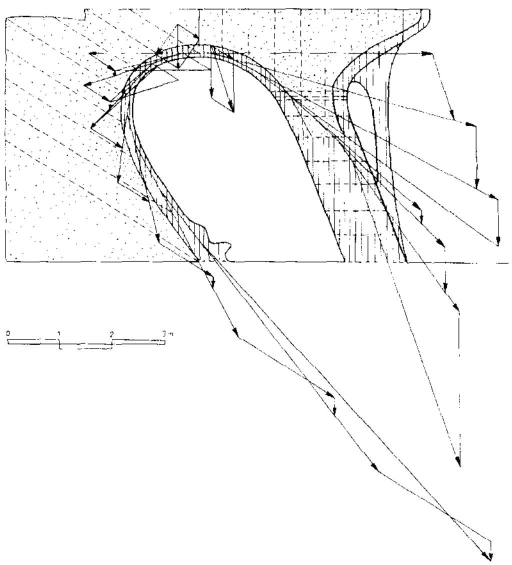


Figure 11
Park Güell (1900–1914). Retaining wall towards Muntanya Pelada. The diagram of forces demonstrates the way Gaudí was determining the most effective shape of retaining walls. (Sweency and Sert 1960, 74)

Load-bearing structures or structural elements that are shaped according to the flow of forces are convincing not only due to their consistency regarding the transfer of loads, but in addition they possess an aesthetic harmony that can positively shape the whole architectural appearance of a building. In view of the formal randomness of shape that prevails in the design of many buildings, even in times of high-tech architecture, the training of both architects and structural engineers has to communicate the capability to derive a sensible design from the function of load transfer. The graphical methods have a special significance in achieving this aim, since they visualise the distribution of forces in the examined structural elements in an excellent way and thus are able to have a positive influence on the design process for load-bearing structures, in the sense of the unity of design, calculation and construction mentioned above.

Analysis of historic structures

One of the tasks of lecturing on structures is the critical examination of historic constructions. Through the graphostatical techniques, students become acquainted with an appropriate tool for analysing, evaluating and preserving historic supporting structures.

Suspended models based on the theory of the thrust line are particularly useful for the analysis of historic vault or arch structures. This gives architecture and structural engineering students the opportunity to examine for example the loadbearing behaviour of masonry domes, without having to acquire deep knowledge of the complex and abstract mathematical techniques of structural vault analysis. However, such an approach is inconceivable without any knowledge of graphostatics and the graphical techniques of representing forces as vectors.

Figure 12 shows a suspended model produced by architecture students at Aachen University for analysing the distribution of forces within the cupola of the cathedral at Aachen (S. Seyedahmadi and T. Daniel. Lehrstuhl für Baugeschichte und Denkmalpflege und Lehrstuhl für Baukonstruktion (Tragwerklehre). Tutors: B. Schindler, R. Gerhardt). The three-dimensional suspended model gave an insight into the annular tension forces acting within the wrought-iron ties inserted into the basis of the vaulted masonry construction. The model also enabled statements to be made about the weight of the stones in the upper zone of the cupola.

Recent applications of graphical methods in engineering practice

With examples from engineering practice, the consulting engineer's office 'Pichler Ingenieure', with contributions by Karen Eisenloffel and Marko Ludwig, demonstrates how graphostatics as a «language of the engineer» (Culmann 1875, 5) can be

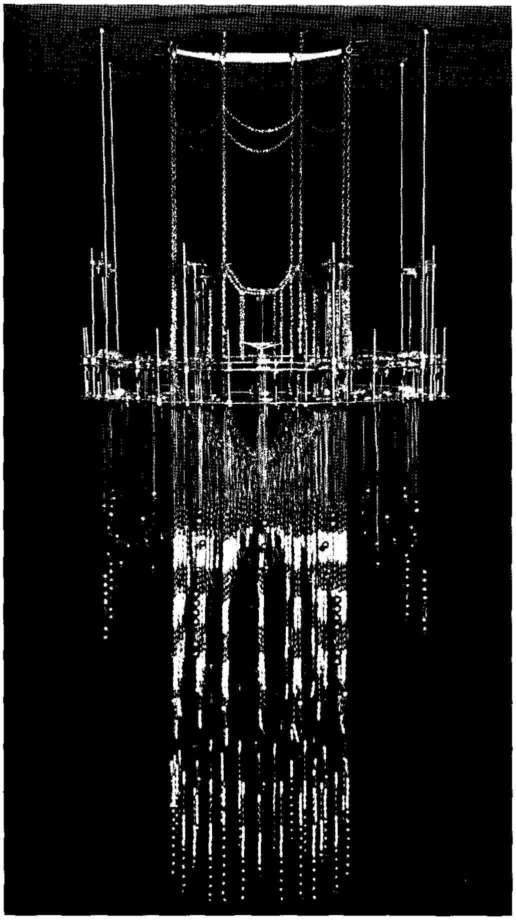


Figure 12
Three dimensional suspended model to analyse the flow of forces in the cupola of the cathedral at Aachen. (Seyedahmadi und Daniel 2002)

utilised even today with the aid of the computer (Pichler et al. 1998). A particularly good example of this approach is the statical examination of the large-span ribbed vault above the main staircase of the city hall «Rotes Rathaus» in Berlin (Figure 13). Cracks had appeared in the plastered brickwork of the ribbed vault, the reasons for which had to be found. The intention was to turn the space above the ribbed vault into a banqueting hall: «We had to investigate whether the damage may have been caused by an overload of the vaults and whether the structure could withstand the envisaged load increase. Due to the type of damage, a steel girder grid to relieve the vault had been considered initially» (Pichler et al. 1998, 227). It appeared sensible to use graphostatics to check anew whether the existing construction would be able to withstand the expected loads. «The force diagrams and thrust lines of the graphical solution were drawn using CAD, so that the iteration steps required for the branched ribbed vault could be carried out without excessive effort and could be checked using a drawing of the existing vault. The result of the first calculations taking into account the additional live load was not surprising. For unilateral loads, the thrust line was outside the vault, i.e. the vault would fail. The advantage of the graphical method —in contrast to spatial computer calculations— is that it indicates a solution through

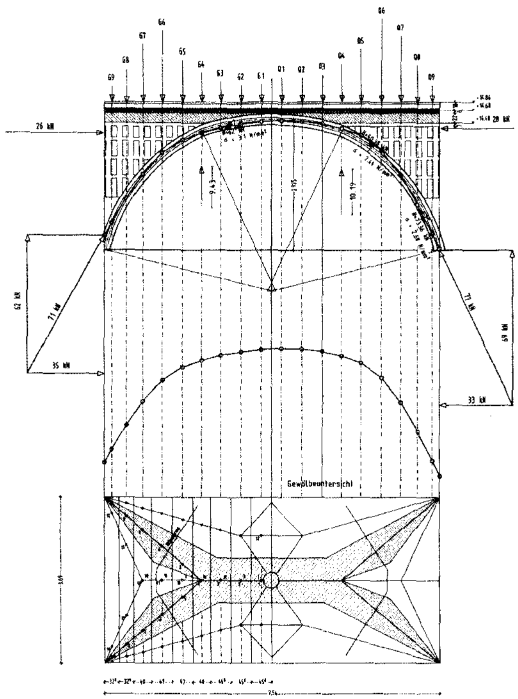


Figure 13
Computer-aided graphostatical investigation of the cross-vault spanning the foyer of the Berlin city hall «Rotes Rathaus». (Eisenloffel and Ermer 2000, 85)

its clarity, namely that increasing the dead loads would prevent the thrust line from leaving the vault. With a ballasting sand layer of 20 cm thickness above the vault, the course of the thrust line could be influenced in such a way that, for a unilateral live load of 3.5 kN/m≈ with a maximum compressive strain of 1.5 N/mm≈, the vault could be shown to be safe.» (Eisenloffel and Ermer 2000, 84–6)

The initially envisaged additional support grid thus became superfluous.

The combination of graphostatics and CAD not only enables the examination of vector and shape-active structures, but also mass-active structures such as floor slabs (Pichler et al. 1998, 229).

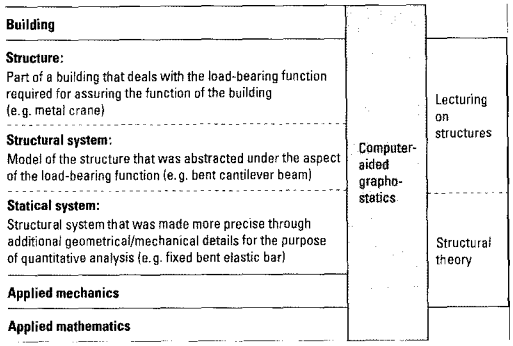


Figure 14
Subject and position of computer-aided graphostatics. (Kurrer 1998, 209)

Outlook

The increasing rationalisation of the engineer's work during the completion phase of the theory of structures (1875–1900) soon lead to the disintegration of graphical statics into respective individual methods, i.e. graphostatics, representing merely graphical recipes. In a development that is comparable with the later introduction of computer statics, graphostatics made structural engineers move away from the design process in so far as graphostatical recipes became a mere means of intellectual techniques, i.e. a technical means or 'techne'. The aesthetic component also faded from the design work of structural engineers. Computer statics reinforced the tendency towards structural design based purely on analytical calculation.

Nevertheless, modern information and communication technology already points towards a systemic design practice that enables structural engineers to regain lost design competency and enables architects to regain lost structural competency on a higher level. One option for moving away from structural design purely based on analytical calculation is the creation of computer-aided graphostatics, positioned at the interface between the practice of structural engineers and of architects in the theory of structures (Figure 14).

Computer-aided graphostatics would have to be designed modularly, so that the user would be able to mobilise each individual structure, e.g. the funicular polygon, through specially developed graphics editors, display it on the screen and combine it with other graphics editors into concrete structures in the sense of graphostatics. Such graphics editors and dimensioning modules based on approximation methods could enable the results of the graphostatical synthesis to be transferred directly to the level of the structural system. Computer-aided graphostatics would thus become the hinge between design and construction. Not least it would be a suitable didactic means to bring together structural engineering and architecture students in the theory of structures.

Further elements could be deduced from the kinematic structural theory developed in the 1880s (Kurrer 2000, 44). For example, the graphical techniques derived from the influence line theory created by Otto Mohr (1835–1918), Emil Winkler (1835–1888), Johann Jakob Weyrauch (1845–1917), Heinrich Müller-Breslau (1851–1925) and in particular Robert Land (1857–1899) during the 1870s and 1880s would provide an appropriate representation for adapted graphics software. Land's theorem would be a nice example for the rediscovery of the visualisation of statics; its graphic representation through the symbolic machine of the computer could significantly increase the 'statical feeling' of the user for the influence of moving effects on the force and displacement parameters.

Computer-aided graphostatics could make the aesthetic component in design work become reality again, since it would facilitate the lost 'statical feeling' at the interface of design and construction becoming second nature again to the user: science would become sensory and poetry would become scientific. It could break the established role behaviour of structural engineers and architects.

Reference list

Culmann, Karl. 1864/1866. *Die graphische Statik.* Zürich: Verlag von Meyer & Zeller.

Culmann, Karl. 1875. *Die graphische Statik.* 2nd ed. Zürich: Meyer & Zeller.

Eisenloffel, Karen and Ingeborg Ermer eds. 2000. *Tragwerkstatt Gerhard Pichler. Entwürfe - Bauten - Konstruktion.* Berlin: Gebr. Mann Verlag.

Faisst, Kurt. 1975. *Veranschaulichung von Tragsystemen durch Modelle. Untersuchungen über den Lernerfolg.* Dissertation. Karlsruhe: Technische Universität Karlsruhe.

Führer, Wilfried; Susanne Ingendaaij and Friedhelm Stein. [1984] 1995. *Der Entwurf von Tragwerken.* Köln: R. Müller.

Gerhardt, Rolf. 1998. Darstellung von Biegemomenten. Ihre experimentelle Ermittlung mit Hilfe der Seillinie. *Baukultur,* 6: 34–37.

Gerhardt, Rolf. 2002a. *Anschauliche Tragwerklehre. Experimentelle Darstellung von Biegemomenten mit Hilfe der Seillinie.* Aachen: Shaker.

Gerhardt, Rolf. 2002b. Reflections on the application of graphical methods to statical problems. In: *Towards a History of Construction,* edited by A. Becchi, M. Corradi, F. Foce and O. Pedemonte, 377–84. Basel/Boston/Berlin: Birkhäuser.

Kurrer, Karl-Eugen. 1998. Baustatik und Ästhetik. Anmerkungen zur Neuausgabe des Werkes «Eisenbauten. Ihre Geschichte und Ästhetik» von Alfred Gotthold Meyer. *Stahlbau,* 67: 205–10.

Kurrer, Karl-Eugen. 2000. Baustatik und Ästhetik. Von der Möglichkeit des Ästhetischen beim statischen Rechnen. *Baukultur,* 8: 38–45.

Kurrer, Karl-Eugen. 2002. *Geschichte der Baustatik.* Berlin: Ernst & Sohn.

Maurer, Bertram. 1998. *Karl Culmann und die graphische Statik.* Berlin/Diepholz/Stuttgart: Verlag für Geschichte der Naturwissenschaft und der Technik.

Pichler, Gerhard; Karen Eisenloffel and Marko Ludwig. 1998. Die graphische Statik in Studium und Praxis. In: *Brückenschläge. Festschrift für Herrn Univ.-Prof. Dr.-Ing. Wilhelm Friedrich Führer zur Vollendung des 60. Lebensjahres,* edited by Lehrstuhl für Baukonstruktion (Tragwerklehre), 223–31. Aachen: Freunde des Reiff, Aachen University.

Poncelet, Jean Victor. 1822. *Traité des propriétés projectives des figures.* Paris.

Ritter, Wilhelm. 1888. *Anwendungen der graphischen Statik. Nach Professor C. Culmann. Erster Teil: Die im Inneren eines Balkens wirkenden Kräfte.* Zürich: Verlag von Meyer & Zeller.

Scholz, Erhard. 1989. *Symmetrie, Gruppe, Dualität. Zur Beziehung zwischen theoretischer Mathematik und Anwendungen in Kristallographie und Baustatik des 19. Jahrhunderts.* Basel: Birkhäuser.

Seyedahmadi, Soheil and Tobias Daniel. 2002. *Geschichte der Konstruktion. Die Kuppel der Aachener Pfalzkapelle.* Studienarbeit Aachen University (RWTH Aachen). Aachen: Lehrstuhl für Baugeschichte und Denkmalpflege and Lehrstuhl für Baukonstruktion (Tragwerklehre).

Stüssi, Fritz. 1951. Karl Culmann und die graphische Statik. *Schweizerische Bauzeitung,* 69: 1–3.

Sweeney, James Johnson and Josep Lluis Sert. 1960. *Antoni Gaudí.* Stuttgart: G. Hatje.

Varignon, Pierre. 1725. *Nouvelle mécanique ou statique.* 2 Vols. Paris: C. Jombert.